GRAYSCALE LEVEL MULTICONNECTIVITY

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Abstract In [5], a novel concept of connectivity for grayscale images was introduced, which is called *grayscale level connectivity*. In that framework, a grayscale image is connected if all its threshold sets below a given level are connected. It was shown that grayscale level connectivity defines a *connection*, in the sense introduced by Jean Serra in [10]. In the present paper, we extend grayscale level connectivity to the case where different connectivities are used for different threshold sets, a concept we call *grayscale level multiconnectivity*. In particular, this leads to the definition of a new operator, called the multiconnected grayscale reconstruction operator. We show that grayscale level multiconnectivity defines a connection, provided that the connectivities used for the threshold sets obey a nesting condition. Multiconnected grayscale reconstruction is illustrated with an example of scale-space representation.

Keywords: Connectivity, Grayscale Images, Reconstruction, Mathematical Morphology Complete Lattice, Scale-Space.

Introduction

In [5], we introduced the notion of *grayscale level connectivity*, in which all threshold sets below a given level k are required to be connected according to a given binary connection. It was shown that a grayscale k-level connected image might have more than one regional maximum, but all regional maxima are at or above level k. It was also shown that grayscale level connectivity can be formulated as a *connection* [10] in an underlattice of the usual lattice of grayscale images. Grayscale level connectivities were shown to lead to effective tools for image segmentation, image filtering and multiscale image representation.

C. Ronse et al. (eds.), Mathematical Morphology: 40 Years On, 129–138.

However, it may be desirable to assign connectivities of varying strictness to different image levels. There may be details of interest at low levels of intensity that can be preserved if one uses a larger connection for lower levels, while at the same time employing a smaller connection at higher levels, in order to prevent relevant regional maxima belonging to different objects from fusing into a single grayscale connected component. In the present paper, we extend the concept of grayscale level connectivity to the case where different binary connections are assigned to different threshold sets of the image. In this case, each threshold set below a given level k is required to be connected according to the respective binary connection. The resulting connectivity is referred to as a *grayscale level multiconnectivity*. We will show that the crucial requirement to be made is that these binary connections be nested. We show that grayscale level multiconnectivity can be formulated as a connection. To compute grayscale level-k multiconnected components, we will employ a novel morphological operator, which we call *multiconnected grayscale reconstruction*; this is an extension of the usual grayscale reconstruction operator [11]. We illustrate the application of multiconnected grayscale reconstruction with an example of *skyline scale-space* [6].

1. Review of Connectivity

We assume that the reader is familiar with basic notions of Lattice Theory and Mathematical Morphology [1, 7]. In this section, we review briefly the theory of connectivity on complete lattices; for a more detailed exposition, please see [10, 4, 2]. Consider a lattice $\mathcal{L}$, with a sup-generating family $\mathcal{S}$. A family $\mathcal{C} \subseteq \mathcal{L}$ is called a *connection* in $\mathcal{L}$ if (i) $O \in \mathcal{C}$, (ii) $\mathcal{S} \subseteq \mathcal{C}$, and ($iii$) if $\{C_\alpha\}$ in $\mathcal{C}$ with $\bigwedge C_\alpha \neq O$, then $\bigvee C_\alpha \in \mathcal{C}$. The family $\mathcal{C}$ generates a *connectivity* on $\mathcal{L}$, and the elements in $\mathcal{C}$ are said to be *connected*. We say that C is a *connected component* of $A \in \mathcal{L}$ if $C \in \mathcal{C}$, $C \leq A$ and there is no $C' \in \mathcal{C}$ different from C such that $C \leq C' \leq A$. In other words, a connected component of an object is a maximal connected part of the object. The set of connected components of A is denoted by $\mathcal{C}(A)$.

We can define an operator $\gamma_x(A)$ on $\mathcal{L}$ that extracts connected components from elements $A \in \mathcal{L}$, by

$$\gamma_x(A) = \bigvee\{C \in \mathcal{C} \mid x \leq C \leq A\},\ x \in \mathcal{S},\ A \in \mathcal{L}. \tag{1}$$

It is easy to see that this operator is increasing, anti-extensive and idempotent, i.e., an opening [7]; it is called the *connectivity opening* associated with $\mathcal{C}$. It can be verified that $\gamma_x(A)$ is the connected component C of A *marked* by x (i.e., such that $x \leq C$).