A NEW DEFINITION FOR THE DYNAMICS

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Abstract We investigate the new definition of the ordered dynamics proposed in [4]. We show that this definition leads to several properties. In particular we give necessary and sufficient conditions which indicate when a transformation preserves the dynamics of the regional maxima. We also establish a link between the dynamics and minimum spanning trees.

Keywords: mathematical morphology, dynamics, graph, watershed, minimum spanning tree

Introduction

The dynamics, introduced by M. Grimaud [1, 2], allows to extract a measure of a regional maximum (or a regional minimum) of a map. Such a measure may be used for eliminating maxima which may be considered as “non significant”. In this paper we investigate a new definition of the dynamics. In particular we establish some equivalence between a transformation which preserves the dynamics and a transformation which preserves some connection values (a kind of measure of contrast) between pairs of points (Th. 11). Such an equivalence is also given with topological watersheds (Prop. 19) and extensions (Prop. 16). Furthermore we establish a link between the dynamics and minimum spanning trees of a graph linking regional maxima, the cost of each arc being precisely the connection value between the corresponding maxima (Th. 14).

1. Basic definitions

Any function may be represented by its different threshold levels [5, 7, 8]. These levels constitute a “stack”. In fact, the datum of a function is equivalent to the datum of a stack. In this section, we introduce definitions for stacks and related notions, this set of definitions allows to handle both the threshold levels of a discrete function and the complements of these levels.

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Discrete maps and stacks

Here and subsequently E stands for a non-empty finite set and K stands for an element of $\mathbb{Z}$, with $K > 0$. If $X \subseteq E$, we write $\overline{X} = \{x \in E \mid x \notin X\}$. If k_1 and k_2 are elements of $\mathbb{Z}$, we define $[k_1, k_2] = \{k \in \mathbb{Z} \mid k_1 \leq k \leq k_2\}$. We set $\mathbb{K} = [-K, +K]$, and $\mathbb{K}^\circ = [-K+1, +K-1]$.

Let $F = \{F[k] \subseteq E \mid k \in \mathbb{K}\}$ be a family of subsets of E with index set $\mathbb{K}$, such a family is said to be a $\mathbb{K}$-*family (on E)*. Any subset $F[k]$, $k \in \mathbb{K}$, is a *section of F (at level k)* or the *k-section of F*. We set:

$$\overline{F} = \{\overline{F}[k] \mid \overline{F}[k] = \overline{F[k]},\ k \in \mathbb{K}\}, \text{ and}$$
$$F^{-1} = \{F^{-1}[k] \mid F^{-1}[k] = F[-k],\ k \in \mathbb{K}\},$$

which are, respectively, the *complement of F* and the *symmetric of F*.

We say that a $\mathbb{K}$-family F is an *upstack on E* if:

$$F[-K] = E,\ F[K] = \emptyset, \text{ and } F[j] \subseteq F[i] \text{ whenever } i < j.$$

We say that a $\mathbb{K}$-family F is a *downstack on E* if:

$$F[-K] = \emptyset,\ F[K] = E, \text{ and } F[i] \subseteq F[j] \text{ whenever } i < j.$$

A $\mathbb{K}$-family is a *stack* if it is either an upstack or a downstack.
We denote by $\mathcal{S}_E^+$ (resp. $\mathcal{S}_E^-$) the family composed of all upstacks on E (resp. downstacks on E). We also set $\mathcal{S}_E = \mathcal{S}_E^+ \cup \mathcal{S}_E^-$.
Let F, G be both in $\mathcal{S}_E^+$ or both in $\mathcal{S}_E^-$. We say that G is *under* F, written $G \subseteq F$ if, for all $k \in \mathbb{K}$, $G[k] \subseteq F[k]$.

Let $F \in \mathcal{S}_E^+$ and let $G \in \mathcal{S}_E^-$. We define two maps from E on $\mathbb{K}$, also denoted by F and G, such that, for any $x \in E$,
$F(x) = \max\{k \in \mathbb{K} \mid x \in F[k]\}$ and $G(x) = \min\{k \in \mathbb{K} \mid x \in G[k]\}$,
which are, respectively, the *functions induced by the upstack F and the downstack G*, $F(x)$ and $G(x)$ are, respectively, the *altitudes of x for F and G*.
Let $F \in \mathcal{S}_E$ and let $x \in E$. We set $S(x, F) = F[k]$, with $k = F(x)$, $S(x, F)$ is the *section of x for F* (see illustration Fig. 1).

Graphs

Throughout this paper, Γ will denote a binary relation on E, which is reflexive and symmetric. We say that the pair (E, Γ) is a *graph*, each element of E is called a *vertex* or a *point*. We will also denote by Γ the map from E to 2^E, such that, for all $x \in E$, $\Gamma(x) = \{y \in E \mid (x, y) \in \Gamma\}$. If $y \in \Gamma(x)$, we say that y is *adjacent to x*. If $X \subseteq E$ and $y \in \Gamma(x)$ for some $x \in X$, we say that *y is adjacent to X*.
Let $X \subseteq E$, a *path in X* is a sequence $\pi = \langle x_0, ..., x_k \rangle$ such that $x_i \in X$, $i \in [0, k]$, and $x_i \in \Gamma(x_{i-1})$, $i \in [1, k]$. We also say that π is a *path from x_0 to x_k in X*. Let x, $y \in X$. We say that x and y are *linked for X* if there exists a path from x to y in X. We say that X is *connected* if any x and y in X are