OPTIMAL SHAPE AND INCLUSION

open problems

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Abstract Access to the shape by its exterior is solved using convex hull. Many algorithms have been proposed in that way. This contribution addresses the open problem of the access of the shape by its interior also called convex skull. More precisely, we present approaches in discrete case. Furthermore, a simple algorithm to approximate the maximum convex subset of star-shaped polygons is described.

Keywords: shape approximation, convex hull, convex skull, potato peeling.

Introduction

In digital image processing we are often concerned with developing specialized algorithms that are dealing with the manipulation of shapes. A very classical and widely studied approach is the computation of the convex hull of an object. However, most of these studies focus only on exterior approaches for the computation of convexity, i.e., they are looking for the smallest convex set of points including a given shape. This contribution addresses the problem of the access of the shape by its interior. The computation of the best shape according to a criterion included in a given one has been studied in many few occasions in the continuous case involving convex skull and potato peeling.

In this article, we present successive configurations of fast approximation of the maximal convex subset of a polygon. First a discrete approach will illustrate an iterative process based on shrinking and convex hull. Second a

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region based approach will be proposed in specific case where the criterion is maximal horizontal-vertical convexity. Third study will be focused to the family of star-shaped polygons P. A simple algorithm extracts the maximal convex subset in $O(k \cdot n)$ in the worst case if n is the size of P and k its number of reflex points.

In section 1, we list classical problems of shape approximation. In section 2, we present the discrete approach followed by h-v convexity in section 3. In section 4, we introduce the Chang and Yap's optimal solution definitions that will be used in the rest of the presentation. In section 5, we present the proposed algorithm based on classical and simple geometric tools for star-shaped polygons. Finally, experiments are given.

1. Shape approximations

In this paper shapes are delimited by polygonal boundary. We suppose that polygons are simple in the sense that they are self-avoiding. Polygon inclusion problems are defined as follows: given a non-convex polygon, how to extract the maximum area subset included in that polygon ? In [7], Goodman calls this problem the *potato-peeling problem*. More generally, Chang and Yap [3] define the polygon *inclusion* problem class $Inc(\mathcal{P}, \mathcal{Q}, \mu)$: given a general polygon $P \in \mathcal{P}$, find the μ-largest $Q \in \mathcal{Q}$ contained in P, where $\mathcal{P}$ is a family of polygons, $\mathcal{Q}$ the set of solutions and μ a real function on $\mathcal{Q}$ elements such that

$$\forall Q' \in \mathcal{Q}, \quad Q' \subseteq Q \Rightarrow \mu(Q') \leq \mu(Q). \tag{1}$$

The maximum area convex subset is an inclusion problem where $\mathcal{Q}$ is the family of convex sets and μ gives the area of a solution Q in $\mathcal{Q}$. The inclusion problem arises in many applications where a quick internal approximation of the shape is needed [2, 5].

In a dual concept, we can mention enclosure problem presented as follows: given a non-convex polygon, how to extract the minimum area subset including that polygon ? We can define the polygon *enclosure* problem class $Enc(\mathcal{P}, \mathcal{Q}, \mu)$: given a general polygon $P \in \mathcal{P}$, find the μ-smallest $Q \in \mathcal{Q}$ containing P, where $\mathcal{P}$ is a family of polygons, $\mathcal{Q}$ the set of solutions and μ a real function on $\mathcal{Q}$ elements such that

$$\forall Q' \in \mathcal{Q}, \quad Q' \supseteq Q \Rightarrow \mu(Q') \geq \mu(Q). \tag{2}$$

Examples

The following list corresponds to examples of various situations depending on specifications of the $\mathcal{P}$ family, the $\mathcal{Q}$ family and the μ measure.

Example 1. $\mathcal{P}$ is a family of simple polygons,