EUCLIDEAN SKELETONS OF 3D DATA SETS IN LINEAR TIME BY THE INTEGER MEDIAL AXIS TRANSFORM

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Abstract A general algorithm for computing Euclidean skeletons of 3D data sets in linear time is presented. These skeletons are defined in terms of a new concept, called the *integer medial axis* (*IMA*) transform. The algorithm is based upon the computation of 3D feature transforms, using a modification of an algorithm for Euclidean distance transforms. The skeletonization algorithm has a time complexity which is linear in the amount of voxels, and can be easily parallelized. The relation of the *IMA* skeleton to the usual definition in terms of centers of maximal disks is discussed.

Keywords: Feature transform, integer medial axis, 3-D Euclidean skeletonization.

1. Introduction

In computer vision, skeleton generation is often one of the first steps in image description and analysis. Intuitively, a skeleton consists of the center lines of an object, and therefore skeletons provide important structural information about image objects by a relatively small number of pixels.

There are four main approaches to skeletonization: 1) thinning, i.e. iterative removal of points from the boundary; 2) wave propagation from the boundary; 3) detection of crest points in the distance transformed image; 4) analytical methods. A large number of skeletonization algorithms exist, see e.g. [15], many of them based upon mathematical morphology [2, 10, 14, 17, 19, 20]. For a parallel 3D skeletonization algorithm based on thinning, see [9].

We note that in algorithms of type 3) one often restricts oneself to local maxima of the distance transform [18], but the resulting skeleton is far from the Euclidean one. The approach we present here is a variant of the third approach, using a definition of skeletons based on Blum's medial axis transform [3].

Often, one is satisfied with approximations to the Euclidean metric (e.g., using chamfer metrics). In 1980, Danielsson [6] gave two good approximating

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Euclidean distance transform algorithms, and applied them to obtain the centers of maximal (integer) disks (*CMD*), see below. He notes (p. 243) that application of skeletons has been hampered by the lack of true Euclidean distance maps. Especially in the 3D case where data size can be very large, many existing algorithms for computing 3D Euclidean skeletons are computationally too expensive [4]. Ge and Fitzpatrick [7] clearly identified the problem to determine the *CMD*: "The problems with existing methods lie in the discrepancies between continuous and discrete image maps". The paper [7] also mentions the goal of linking the centers of maximal disks into *connected* skeletons.

The main contribution of the present work is that we present a simple and easily parallelizable linear time algorithm which computes a skeleton defined in terms of a new concept, called the *integer medial axis* (*IMA*) transform. The algorithm works in arbitrary dimensions, and is based upon the general linear time Euclidean distance transform (EDT) algorithm of Hirata [8], which has been rediscovered several times, i.e., by ourselves, see Meijster *et al.* [13], and later by Maurer *et al.* [11, 12]. The skeletonization algorithm has two phases. First, a feature transform is computed, which uses essentially the same algorithm as for the distance transform, the difference being that not only distances are computed, but also the boundary points which realize the closest distance. The actual skeletonization is performed in a second pass through the data, where the integer medial axis is computed by assigning points to the skeleton depending on their feature transform.

Our method does not aim at a minimal skeleton useful for image compression with exact reconstruction, but at a computation of connected skeletons directly from the Euclidean feature transform, thus avoiding the costly and complicated phase of removing centers of not-quite-maximal disks by the techniques of [16]. We establish a number of mathematical properties of the *IMA* and point out some relations to Blum's real medial axis (*RMA*) and to the *CMD* skeleton. More work is needed to establish the topological characteristics of the *IMA* skeleton.

Often, simplification or pruning of the skeleton is used as a postprocessing step to remove unwanted points, which arise especially in noisy data [1]. In our approach, skeleton pruning can be handled in the algorithm itself, by a single adjustable parameter through which one can prune the skeleton during the second pass of the algorithm.

In order to derive our algorithm, we first modify the *EDT* algorithm of Meijster *et al.* to calculate 3D feature transforms, from which the *IMA* skeletons are derived. For all program parts, explicit and compact pseudocode is given.