DIGITIZATION OF NON-REGULAR SHAPES

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Abstract Only the very restricted class of r-regular shapes is proven not to change topology during digitization. Such shapes have a limited boundary curvature and cannot have corners. In this paper it is shown, how a much wider class of shapes, for which the morphological open-close and the close-open-operator with an r-disc lead to the same result, can be digitized correctly in a topological sense by using an additional repairing step. It is also shown that this class is very general and includes several commonly used shape descriptions. The repairing step is easy to compute and does not change as much pixels as a preprocessing regularization step. The results are applicable for arbitrary, even irregular, sampling grids.

Keywords: shape, digitization, repairing, topology, reconstruction, irregular grid

Introduction

The processing of images by a computer requires their prior digitization. But as Serra already stated in 1980 [5], “To digitize is not as easy as it looks.” Shapes can be regarded as binary images and the simplest model for digitization is to take the image information only at some sampling points and to set the associated pixels to these values. Unfortunately even for this simple digitization model only a very restricted class of binary images is proven to preserve topological characteristics during digitization: Serra proved that the homotopy tree (i.e. the inclusion properties of foreground- and background components) of r-regular images (see Definition 1) does not change under digitization with a hexagonal grid of certain density [5]. Similarily Pavlidis showed that such images can be digitized with square grids of certain density without changing topology [4]. Latecki [3] also referred to this class of shapes. Recently the author proved together with Köthe that r-regular sets are not only sufficient but also necessary to be digitized topologically correctly with *any* sampling grid of a certain density [6]. But most shapes are not r-regular, e.g. have corners. To solve this problem Pavlidis said “Indeed suppose that we have a class of objects whose contours contain corners. We may choose a radius of curvature

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r and replace each corner by a circular arc with radius r" [4]. This approach to make shapes r-regular has two problems: (1) Pavlidis gives no algorithm how to do it exactly. He also does not say, for which shapes it is possible without changing the topology of the set. (2) It is a preprocessing step and thus cannot be computed by a computer, which only gets the digitized information. The aim of this paper is to solve both problems. After a short introduction in the definitions of r-regular images, sampling and reconstruction (section 1), the class of r-halfregular sets is defined in section 2, whose elements can be converted into r-regular sets by using a very simple morphological preprocessing step. In order to solve the second problem, it is shown how these shapes can be digitized topologically correctly by using a postprocessing algorithm instead of the preprocessing. These results are applicable for digitization with any type of sampling grid – only a certain density is needed. In section 3 it is shown that the concept of r-halfregular shapes includes several other shape descriptions. Finally in section 4 the postprocessing step is even more simplified in case of certain sampling grids. For square grids it simply means to delete all components and to fill all holes, which do not contain a 2x2 square of pixels. This is remarkably similar to the results of Giraldo et al. [2], who proved that finite polyhedra can be digitized with intersection digitization without changing their homotopy properties by filling all holes, which do not contain a 2x2 square. Unfortunately their approach was not applicable to other sets and was restricted to another digitization model.

1. Regular Images, Sampling and Reconstruction

At first some basic notations are given: The Euclidean distance between two points x and y is noted as $d(x, y)$ and the Hausdorff distance between two sets is the maximal distance between one point of one set and the nearest point of the other. The Complement of a set A will be noted as A^c. The boundary ∂A is the set of all common accumulation points of A and A^c. A set A is open, if it does not intersect its boundary and it is closed if it contains the boundary, $A^0 := A \setminus \partial A$, $\overline{A} := A \cup \partial A$. $\mathcal{B}_r(c) := \{x \in \mathbb{R}^2 | d(x, c) \leq r\}$ and $\mathcal{B}_r^0(c) := (\mathcal{B}_r(c))^0$ denote the closed and the open disc of radius r and center c. If $c = (0, 0)$, write $\mathcal{B}_r$ and $\mathcal{B}_r^0$. The r-dilation $A \oplus \mathcal{B}_r^0$ of a set A is the union of all open r-discs with center in A and the r-erosion $A \ominus \mathcal{B}_r^0$ is the union of all center points of open r-discs lying inside of A. The morphological opening with an open r-disc is defined as $A \circ \mathcal{B}_r^0 := (A \ominus \mathcal{B}_r^0) \oplus \mathcal{B}_r^0$ and the respective closing as $A \bullet \mathcal{B}_r^0 := (A \oplus \mathcal{B}_r^0) \ominus \mathcal{B}_r^0$. The concept of r-regular images was introduced independently by Serra [5] and Pavlidis [4]. These sets are extremely well behaved – they are smooth, round and do not have any cusps (e.g. see Fig. 2B). Furthermore r-regular sets are invariant under morphological opening and closing, as already stated by Serra [5].