MATHEMATICAL MODELING OF THE RELATIONSHIP "BETWEEN" BASED ON MORPHOLOGICAL OPERATORS

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Abstract The spatial relationship "between" is a notion which is intrinsically both fuzzy and contextual, and depends in particular on the shape of the objects. The few existing definitions do not take into account these aspects. We propose here definitions which are based on morphological operators and a fuzzy notion of visibility in order to model the main intuitive acceptions of the relation. We distinguish between cases where objects have similar spatial extensions and cases where one object is much more extended than the other. These definitions are illustrated on real data from brain images.

Keywords: Relationship "between", spatial reasoning, fuzzy dilation, visibility.

Introduction

Spatial reasoning and structural object recognition in images rely on characteristics or features of objects, but also on spatial relationships between these objects, which are often more stable and less prone to variability. Several of these relationships (like set theoretical ones, adjacency, distances) are mathematically well defined. Other ones are intrinsically vague and imprecise. Their modeling in the framework of fuzzy sets proved to be well adapted. This is the case for instance for directional relative direction, which can be adequately defined using fuzzy directional dilations [2]. Interestingly enough, these basic

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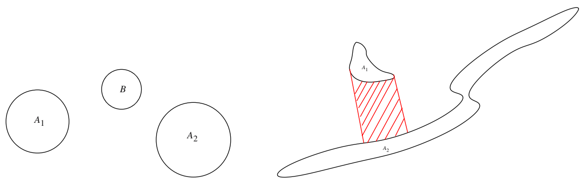
relationships can be expressed in terms of mathematical morphology, which endows this framework with a unifying feature [3]. More complex relationships have received very little attention until now. In this paper, we deal with the "between" relation, and propose to model it based on simple morphological operators.

Definitions of "between" in dictionaries involve the notion of separation ("in the space that separates"). From a cognitive point of view, two factors appear to play a role: the convex hull of the union of both objects and the notion of visibility. Several difficulties arise when trying to model this relationship. First, it is intrinsically vague and imprecise, even if objects are precise. For instance, in Figure 1 (a), we would like to consider that B is not completely between A_1 and A_2 but that it is between them to some degree. Moreover, the relation has several meanings and may vary depending on shape. The definitions should therefore be contextual rather than absolute. For instance, the between relation cannot be defined in the same way whether the objects have similar spatial extensions or not (Figure 1 (b)), hence the necessary dependence on the context of the definitions. The semantics of "between" change depending on whether we consider a person between two buildings, a fountain between a house and a road, or a road passing between two houses. These differences have been exhibited in cognitive and linguistic studies [13].



(a) (b)

Figure 1. (a) Is the object B between A_1 and A_2 and to which degree? (b) An example of objects with different spatial extension where a contextual definition is appropriate [13].

The primary aim of this paper is to propose some definitions of the relationship "between", modeling mathematically these intuitive ideas. More precisely, we try to answer the following question: *Which is the region of space, denoted by $\beta(A_1, A_2)$, located between two objects A_1 and A_2?* From the answer, we can then assess the degree to which an object B is between A_1 and A_2 by defining an appropriate measure of comparison between $\beta(A_1, A_2)$ and B [4].

Although this problem received very little attention, it was addressed by different communities. Approaches found in the domain of spatial logics and qualitative spatial reasoning rely on colinearity between points [1] or between centers of spheres [11]. They do not take into account the shape of objects nor the fuzziness of the "between" relationship. To our knowledge, only two