NUMERICAL RESIDUES

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Abstract

Binary morphological transformations based on the residues (ultimate erosion, skeleton by openings, etc.) are extended to functions by means of the transformation definition and of its associated function based on the analysis of the residue evolution in every point of the image. This definition allows to build not only the transformed image itself but also its associated function, indicating the value of the residue index for which this evolution is the most important. These definitions are totally compatible with the existing definitions for sets. Moreover, they have the advantage of supplying effective tools for shape analysis on one hand and, on the other hand , of allowing the definition of new residual transforms together with their associated functions. Two of these numerical residues will be introduced, called respectively ultimate opening and quasi-distance and, through some applications, the interest and efficiency of these operators will be illustrated.

1. Introduction

In binary morphology there are some operators based on the detection of residues of parametric transformations. Among these operators, the ultimate erosion or the skeleton by maximal balls can be quoted. They can more or less easily be extended to greytone images. These extensions are however of little use because it is difficult to exploit them. This paper explains the reasons of this difficulty and proposes a means to obtain interesting information from these transformations. It also introduces new residual transformations and illustrates their use in applications.

2. Binary residues: reminder of their definition

Only operators corresponding to the residues of two primitive transforms will be addressed here. A residual operator θ on a set X is defined by means

C. Ronse et al. (eds.), Mathematical Morphology: 40 Years On, 23–32.

of two families of transformations (the primitives) depending on a parameter i, $(i \in I)$, ϕ_i and ζ_i, with $\phi_i \geq \zeta_i$. The residue of size i is the set: $r_i = \phi_i / \zeta_i$, the transformation θ is then defined as: $\theta = \cup_{i \in I} r_i$

Usually, ϕ_i is an erosion ϵ_i. According to the choice of ζ_i, we get the different following operators:

- The ultimate erosion [2]; the operator ζ_i is then the elementary opening by reconstruction of the erosion ϵ_i: $\zeta_i = \gamma_{rec}(\epsilon_i)$
- The skeleton by maximal balls; in that case the operator ζ_i is the elementary opening of the erosion of size i: $\zeta_i = \gamma(\epsilon_i)$

Generally a function q, called residual or associated function is linked to these transformations. The support of q is the transformed θ(X) itself. This function takes in every point x, the value of index i of residue r_i containing point x (or more exactly the value i+1, so that this function is different from zero for r_0). Indeed, in the binary case, if the primitives are correctly chosen, to every point x corresponds a unique residue. One has then:

$$q(x) = i\ + 1 :\ x \in r_i$$

For the ultimate erosion, this function corresponds to the size of the ultimate components. For the skeleton, it is called *quench function* and corresponds to the size of the maximal ball centered in x.

3. Extension to greytone images

It is common to read or to hear that these operators can be extended without any problem to the numerical case (greytone images). It is just as remarkable to notice that there is practically no interesting application of these operators in greytone image analysis. Two factors explain this established fact:

- Extension is maybe not "as evident" as it appears to be, for the transformation θ itself but also and especially for the associated function q.
- The amount of information is often too excessive and little relevant, a fact which does not ease the use of these numerical transformations.

Definition of the operator θ in the numerical case

A "simple" definition of θ can be written:

$$\theta = \sup_{i \in I} (\psi_i - \zeta_i)$$

by using the numerical equivalents of the set union and difference operators.

However by doing so, a first problem appears. The subtraction of functions is not really equivalent to the set difference. In the binary case, we had, for a