STOCHASTIC SHAPE OPTIMISATION

Costin Alin Caciu, Etienne Decencière and Dominique Jeulin
Centre de Morphologie Mathématique, Ecole des Mines de Paris
35, rue Saint Honoré, 77300 Fontainebleau, France
Costin.Caciu@cmm.ensmp.fr

Abstract We present a constrained shape optimisation problem solved via metaheuristic stochastic techniques. Genetic Algorithms are briefly reviewed and their adaptation to surface topography optimisation is studied. An application to flow optimisation issues is presented.

Keywords: Shape, Topography, Stochastic Optimisation, Genetic Algorithms

Introduction

Geometrical shape or surface topography optimisations are quite numerous among the optimisation problems, since they are encountered and play often an important role in most technical domains. Traditionally, deterministic models or expensive experimental tests are employed to solve these issues.

Computing power has grown considerably in the past years, dramatically promoting the interest in numerical techniques. Numerical simulations and models become issues for problems in many domains. They opened the path, serving as objective functions, to a large number of stochastic optimisation techniques working on discretised search spaces.

We will focus here on *Genetic Algorithms* (GA) [1, 4]. The main advantage of this numerical optimisation approach, compared to deterministic ones, is its extended search space and its robustness. In return, it requires complex and onerous computations, and its convergence rate is particulary slow, which represents its main weakness.

We will first describe (Section 1) the precise problem that led us to this study. Section 2 gives a tutorial introduction to stochastic optimisation and in Section 3 we describe the GAs' adaptation to our specific problem. Section 4 exposes the results of GAs' application for a particular fitness function. A summary and the conclusion are presented in Section 5.

C. Ronse et al. (eds.), Mathematical Morphology: 40 Years On, 333–342.

1. Problem

Let us assume we have to optimise a shape $\mathcal{S}$ representing an opened volume (Figure 1) in a smooth surface $\mathcal{P}$. The optimisation is controlled by a fitness function f and restricted by several constraints $\mathcal{C}$:

- $\mathcal{S}$ is a connected shape.
- $\mathcal{S}$ has a constant volume V and a constant opened surface S_s.
- the depth in any point of $\mathcal{S}$ may vary between d_{min} and d_{max}.

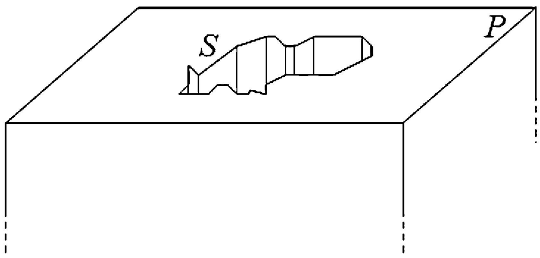

Figure 1. 3D shape to optimise

To build a model as general as possible, the fitness function f may be any measure on $\mathcal{S}$ respecting $\mathcal{C}$.

For the problem above, the implementation of a deterministic optimisation is not an option. The shape $\mathcal{S}$ may take complex forms, impossible to define analytically; likewise, the fitness function f may be also difficult to estimate analytically; moreover, in many situations, the complexity of the employed fitness function may lead to unpredictable deterministic relations between $\mathcal{S}$ and f, impossible to pursuit. In consequence, in order to escape these difficulties and to preserve a wide space in search for improvement, we have to work with discretised shapes and numerical fitness functions, and employ appropriate *stochastic optimisation methods*.

A coarse discretisation of a random surface shape $\mathcal{S}$ is presented in Figure 2(a). In this 2D image representation the depth of the opening is coded by the grey levels of pixels (8 bits), the white background being assigned to $\mathcal{P}$ and the grey pixels to the various depths of $\mathcal{S}$. A three-dimensional representation of the previous image is illustrated in Figure 2(b).