ON THE LOCAL CONNECTIVITY NUMBER OF STATIONARY RANDOM CLOSED SETS

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Abstract Random closed sets (RACS) in the d–dimensional Euclidean space are considered, whose realizations belong to the extended convex ring. A family of nonparametric estimators is investigated for the simultaneous estimation of the vector of all specific Minkowski functionals (or, equivalently, the specific intrinsic volumes) of stationary RACS. The construction of these estimators is based on a representation formula for the expected local connectivity number of stationary RACS intersected with spheres, whose radii are small in comparison with the size of the whole sampling window. Asymptotic properties of the estimators are given for unboundedly increasing sampling windows. Numerical results are provided as well.

Keywords: Mathematical morphology; random closed sets; stationarity; Minkowski functionals; intrinsic volumes; nonparametric estimation; local Euler–Poincaré characteristic; principal kinematic formula; Boolean model

Introduction

The theory of random closed sets (RACS) and its morphological aspects with emphasis on applications to image analysis have been developed in the second half of the 20th century. This scientific process has been significantly influenced by the pioneering monographs of G. Matheron [6] and J. Serra [15, 16]. It turned out that *Minkowski functionals* or, equivalently, *intrinsic volumes* are important characteristics in order to describe binary images, since they provide useful information about the morphological structure of the underlying RACS. In particular, the so–called *specific intrinsic volumes* of stationary RACS have been intensively studied for various models from stochastic geometry.

There exist several approaches to the construction of statistical estimators for particular specific intrinsic volumes of stationary RACS in two and three

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dimensions. However, in many cases, only little is known about goodness properties of these estimators, like unbiasedness, consistency, or distributional properties. Furthermore, an extra algorithm has to be designed for the estimation of each specific intrinsic volume separately.

In contrast to this situation, the method of moments proposed in the present paper provides a unified theoretical and algorithmic framework for simultaneous nonparametric estimation of all specific intrinsic volumes, in an arbitrary dimension $d \geqslant 2$. The construction principle of these estimators, which is similar to the approach considered in [11], is based on a representation formula for the (expected) local connectivity number of stationary RACS intersected with spheres, whose radii are small in comparison with the size of the whole sampling window. It can be considered as a statistical counterpart to a method for the simultaneous computation of all intrinsic volumes of a deterministic polyconvex set based on the principal kinematic formula.

Our estimators are unbiased by definition. Moreover, under suitable integrability and mixing conditions, they are mean–square consistent and asymptotically normal distributed. This can be used in order to establish asymptotic tests for the vector of specific intrinsic volumes.

Notice that the method of moments (which is also called the method of intensities by some authors) has been used in the analysis of various further statistical aspects of models from stochastic geometry, for example, in order to estimate the intensity of germs and other characteristics of the Boolean model; see e.g. [7], and Sections 5.3–5.4 in [13].

The present paper is organized as follows. Some necessary preliminaries on Minkowski functionals and intrinsic volumes, respectively, are given in Section 1. In Section 2, the computation of intrinsic volumes of deterministic polyconvex sets is briefly discussed. The above–mentioned representation formula for the (expected) local connectivity number of stationary RACS is stated in Section 3; see Proposition 3.1. We give an alternative proof of this representation formula which makes use of an explicit extension of Steiner's formula for convex bodies to the convex ring. The result of Proposition 3.1 is then used in Section 4 in order to construct a family of nonparametric estimators for all $d + 1$ specific intrinsic volumes simultaneously. The construction principle of these estimators is described and their asymptotic properties are discussed. A related family of least–squares estimators is also provided in Section 4. In Section 5, some aspects of variance reduction using *kriging of the mean* are touched upon. Finally, in Section 6 numerical results are given for the planar Boolean model with spherical primary grains. They are compared with those obtained by another method described in [10] for the computation of specific intrinsic volumes.