MORPHOLOGY FOR HIGHER-DIMENSIONAL TENSOR DATA VIA LOEWNER ORDERING

Bernhard Burgeth,[1] Nils Papenberg, [1] Andres Bruhn, [1] Martin Welk, [1] Christian Feddern, [1] and Joachim Weickert [1]

[1] *Saarland University*
Faculty of Mathematics and Computer Science
Bld. 27.1, P.O. Box 15 11 50, 66041 Saarbruecken
Germany
{burgeth,papenberg,bruhn,welk,feddern,weickert}@mia.uni-saarland.de

Abstract The operators of greyscale morphology rely on the notions of maximum and minimum which regrettably are not directly available for tensor-valued data since the straightforward component-wise approach fails.
This paper aims at the extension of the maximum and minimum operations to the tensor-valued setting by employing the Loewner ordering for symmetric matrices. This prepares the ground for matrix-valued analogs of the basic morphological operations. The novel definitions of maximal/minimal matrices are rotationally invariant and preserve positive semidefiniteness of matrix fields as they are encountered in DT-MRI data. Furthermore, they depend continuously on the input data which makes them viable for the design of morphological derivatives such as the Beucher gradient or a morphological Laplacian. Experiments on DT-MRI images illustrate the properties and performance of our morphological operators.

Keywords: Mathematical morphology, dilation, erosion, matrix-valued images, diffusion tensor MRI, Loewner ordering

Introduction

A fruitful and extensive development of morphological operators has been started with the path-breaking work of Serra and Matheron [11, 12] almost four decades ago. It is well documented in monographs [8, 13–15] and conference proceedings [7, 16] that morphological techniques have been successfully used to perform shape analysis, edge detection and noise suppression in numerous applications. Nowadays the notion of image also encompasses tensor-valued data, and as in the scalar case one has to detect shapes, edges and eliminate noise. This creates a need for morphological tools for matrix-valued data.

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Matrix-valued concepts, that truly take advantage of the interaction of the different matrix-channels have been developed for median filtering [20], for active contour models and mean curvature motion [5], and for nonlinear regularisation methods and related diffusion filters [17, 19]. In [4] the basic operations dilation and erosion as well as opening and closing are transfered to the matrix-valued setting at least for 2×2 matrices. However, the proposed approaches lack the continuous dependence on the input matrices which poses an insurmountable obstacle for the design of morphological derivatives.

The goal of this article is to present an alternative and more general approach to morphological operators for tensor-valued images based on the Loewner ordering. The morphological operations to be defined should work on the set $\mathrm{Sym}(n)$ of symmetric $n \times n$ matrices and have to satisfy conditions such as:

(i) Continuous dependence of the basic morphological operations on the matrices used as input for the aforementioned reasons,

(iii) preservation of the positive semidefiniteness of the matrix field since DT-MRI data sets posses this property,

(iii) rotational invariance.

It is shown in [4] that the requirement of rotational invariance already rules out the straightforward component-wise approach. In this paper we will introduce a novel notion of the minimum/maximum of a finite set of symmetric matrices which will exhibit the above mentioned properties.

The article has the following structure: The subsequent section gives a brief account of the morphological operations we aim to extend to the matrix-valued setting. In section 3 we present the crucial maximum and minimum operations for matrix-valued data based on the Loewner ordering. We report the results of our experiments with various morphological operators applied to real DT-MRI images in section 4. The last section 5 provides concluding remarks .

1. Morphological Operators

Standard morphological operations utilise the so-called *structuring element* to work on images represented by scalar functions $f(x, y)$ with $(x, y) \in I\!R^2$. Greyscale *dilation* $\oplus$, resp., *erosion* $\ominus$ w.r.t. B is defined by

$$\begin{aligned} (f \oplus B)(x,y) &:= \sup\,\{f(x-x',\, y-y') \mid (x',y')\in B\}, \\ (f \ominus B)(x,y) &:= \inf\,\{f(x-x',\, y-y') \mid (x',y')\in B\}. \end{aligned}$$

The combination of dilation and erosion gives rise to various other morphological operators such as *opening* and *closing*,

$$f \circ B := (f \ominus B) \oplus B\,, \qquad f \bullet B := (f \oplus B) \ominus B\,,$$