EFFICIENT PATH OPENINGS AND CLOSINGS

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To Henk.

Abstract Path openings and closings are algebraic morphological operators using families of thin and oriented structuring elements that are not necessarily perfectly straight. These operators are naturally translation invariant and can be used in filtering applications instead of operators based on the more standard families of straight line structuring elements. They give similar results to area or attribute-based operators but with more flexibility in the constraints.

Trivial implementations of this idea using actual suprema or infima of morphological operators with paths as structuring elements would imply exponential complexity. Fortunately a linear complexity algorithm exists in the literature, which has similar running times as an efficient implementation of algebraic operators using straight lines as structuring elements.

However even this implementation is sometimes not fast enough, leading practitioners to favour some attribute-based operators instead, which in some applications is not optimal.

In this paper we propose an implementation of path-based morphological operators which is shown experimentally to exhibit logarithmic complexity and comparable computing times with those of attribute-based operators.

Keywords: Algebraic morphological operators, attributes, complexity.

Introduction

Many problems in image analysis involve oriented, thin, line-like objects, for example fibres [22, 20], hair [19, 15], blood vessels [8], grid lines on stamped metal pieces [21] and others.

For bright and elongated structures, a common approach for detecting these features is to use to use an infimum of openings using lines as structuring el-

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ements oriented in many directions [10]. The result is an isotropic operator if the line structuring element lengths are adjusted to be independent of orientation [11]. Recursive implementations of openings at arbitrary angles have been proposed and yield linear-time algorithms [17] with respect to the length of the structuring elements. If desired features are very thin, a translation-invariant algorithm should be used [18].

Area and attributes openings [1, 12, 23] are also often used for the analysis of thin structures. An area opening of parameter λ is equivalent to the supremum of all the openings by connected structuring elements of area λ. Obviously this includes all the straight line structuring elements of this length. Practitioners often note that using only straight line structuring elements removes too much of the desired features, while using area operators does not allow them to distinguish between long and narrow features on the one hand, and short compact ones on the other. It is sometimes, but not always possible to combine these operators, or to use morphological reconstruction to obtain the desired outcome.

Recently efficient morphological operators equivalent to using families of narrow, elongated but not necessarily perfectly straight structuring elements were proposed in [3] and [6], together with an algorithm for computing the operator with linear complexity with regards to the length of the structuring elements. These path operators constitute a useful medium between operators using only straight lines and those using area or other attributes.

In the remainder we propose a significantly faster algorithm for implementing path operators, with observed logarithmic complexity with respect to the length of the structuring elements.

1. Path-based morphological opening

The theory of path openings is explained in detail in [6] and in a shorter fashion in [5]. We only summarize the main points here.

Adjacency and paths

Let E be a discrete 2-D image domain, a subset of $\mathbb{Z}^2$. Then $\mathcal{B} = \mathcal{P}(E) = 2^E$ is the set of binary images and $\mathcal{G} = \mathcal{T}^E$ the space of grey-scale functions, where $\mathcal{T}$ is the set of grey values. We assume E is endowed with an adjacency relation $x \mapsto y$ meaning that there is a directed edge going from x to y. Using the adjacency relation we can define the dilation $\delta(\{x\}) = \{y \in E, x \mapsto y\}$. The L-tuple $\boldsymbol{a} = (a_1, a_2, ..., a_L)$ is called a *δ-path of length L* if $a_{k+1} \in \delta(\{a_k\})$ for $k = 1, 2, ..., L-1$. Given a path $\boldsymbol{a}$ in E, we denote by $\sigma(\boldsymbol{a})$ the set of its elements, i.e: $\sigma(a_1, a_2, ..., a_L) = \{a_1, a_2, ..., a_L\}$. We denote the set of all δ-paths of length L by Π_L, and the set of δ-paths of length L contained in a subset X of E is denoted by $\Pi_L(X)$.