SECOND-ORDER CONNECTED ATTRIBUTE FILTERS USING MAX-TREES

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Abstract The work presented in this paper introduces a novel method for second-order connected attribute filtering using Max-Trees. The proposed scheme is generated in a recursive manner from two images, the original and a modified copy by an either extensive or an anti-extensive operator. The tree structure is shaped by the component hierarchy of the modified image while the node attributes are based on the connected components of the original image. Attribute filtering of second-order connected sets proceeds as in conventional Max-Trees with no further computational overhead.

Keywords: second-order connectivity, Max-Tree, attribute filters, clustering, partitioning

Introduction

The concept of second-order connectivity [6, 8] is a generalization of conventional connectivity summarizing two perceptual conditions known as *clustering* and *partitioning*. In brief, when clustering objects close enough to each other in morphological terms, are considered as a single entity, while when partitioning isolated object regions interconnected by thin elongated segments are handled as independent objects. The theoretic framework developed to formalize this [8, 1] defines the two cases by means of connected openings that consider the intersection of the original image with the generalized connectivity map. Extensions to a multi-scale approach employing a hierarchical representation of connectivity have also been made. Two examples are connectivity pyramids [2] and *Connectivity Tree* [9], which quantify how strongly or loosely objects or object regions are connected.

Algorithmic realizations of this framework originally suggested the use of generalized binary and gray-scale reconstruction operators [1] for recovering the object clusters or partitions. This introduced a family of filters based on topological object relations with width as the attribute criterion. Efficient al-

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gorithms for the more general class of gray-scale attribute filters using second-order connectivity have not yet been proposed. In this paper we will present a method based on Max-Trees [7].

Our method builds a hierarchical representation based on gray scale image pairs comprising the original image and a modified copy by an increasing and either extensive or anti-extensive operator. The algorithm, referred to as *Dual Input Max-Tree* is inspired by [7, 10] and demonstrates an efficient way of computation of generalized area openings . The results extend easily to other attribute filters.

A presentation of our method is given in this paper which is organized as follows: The first section gives a brief overview of the concept of connectivity and attribute filters. A short description of second-order connectivities follows in the second section where the two cases of clustering and partitioning are described in a connected opening form. A review of the Max-Tree algorithm is given in the third section complemented by a description of our implementation while results and conclusions are discussed in the fourth section.

1. Connectivity and Connected Filters

This section briefly outlines the concept of connectivity from the classical morphological prospective. For the purpose of this analysis we assume a universal (non-empty) set E and we denote by $\mathcal{P}(E)$ the collection of all subsets of E. A set X representing a binary image such that $X \subseteq E$ is said to be connected if it cannot be partitioned into two non-empty closed or opened sets. Expressing this using the notion of *connectivity classes*, Serra [8] derived the following definition:

DEFINITION 1 *A family $\mathcal{C} \subseteq \mathcal{P}(E)$ with E an arbitrary non-empty set, is called a connectivity class if it satisfies:*

1 $\emptyset \in \mathcal{C}$ and $\{x\} \in \mathcal{C}$ for $x \in E$,

2 if $C_i \in \mathcal{C}$ with $i = 1, ...I$ and $\bigcap_{i=1}^{N} C_i \neq \emptyset$, then $\bigcup_{i \in I} C_i \in \mathcal{C}$

where $\{x\}$ denotes a singleton.

The class $\mathcal{C}$ in this case defines the connectivity on E and any subset of $\mathcal{C}$ is called a *connected set* or a *connected component*.

Given the connected sets $C_x \in \mathcal{C}$ containing $x \in X$, the connected opening connected, opening Γ_x can be expressed as the union of all C_x:

$$\Gamma_x(X) = \bigcup \{C_x \in \mathcal{C} | x \in C_x \text{ and } C_x \subseteq X\} \tag{1}$$

With all sets C_x containing at least one point of X in their intersection, i.e. x, their union $\Gamma_x(X)$ is also connected. Furthermore $\forall x \notin X, \Gamma_x(X) = \emptyset$.