ATTRIBUTE-SPACE CONNECTED FILTERS

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Abstract In this paper connected operators from mathematical morphology are extended to a wider class of operators, which are based on connectivities in higher dimension spaces, similar to scale spaces which will be called attribute spaces. Though some properties of connected filters are lost, granulometries can be defined under certain conditions, and pattern spectra in most cases. The advantage of this approach is that regions can be split into constituent parts before filtering more naturally than by using partitioning connectivities.

Keywords: Mathematical morphology, multi-scale analysis, connected filters, perceptual grouping.

1. Introduction

Semantic analysis of images always involves grouping of pixels in some way. The simplest form of grouping is modelled in digital image processing by connectivity [4], which allows us to group pixels into connected components or flat-zones in the grey-scale case. In mathematical morphology, *connected operators* have been developed which perform filtering based on these kinds of groupings [7][8][9]. However, the human observer may either interpret a single connected component of a binary image as multiple visual entities, or group multiple connected components into a single visual entity. These properties have to some extent been encoded in second-order connectivities, which can be either partitioning or clustering [1] [3][12].

In this paper I will demonstrate a problem with partitioning connectivities when used for second-order connected attribute filters, due to the large numbers of singletons they produce in the image. This *over-segmentation* effect is shown in Fig. 1. It will be shown that these attribute filters reduce to performing e.g. an opening with ball B followed by an application of the attribute filter using the normal (4 or 8) connectivity. The approach presented here is is different from second-order connectivities, in that it restates the connectiv-

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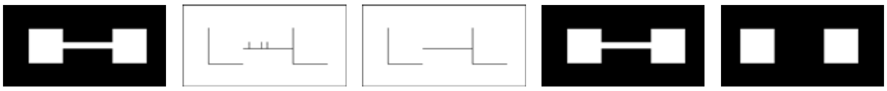

(a) (b) (c) (d) (e)

Figure 1. Attribute-space compared to regular attribute filtering: (a) original image X; (b) the connected components of X according to $\mathcal{C}^{\psi}$, with ψ an opening by a 3×3 structuring element (see Section 3); (d) partitioning of X by attribute space method of Section 4; (e) regular attribute thinning $\Psi^{T}_{\psi}(X)$ with $T(C) = (I(C)/A^2(C) < 0.5)$; (f) attribute-space connected attribute thinning $\Psi^{T}_{A}(X)$ with the same T. T is designed to remove elongated structures. Note that only the attribute-space method removes the elongated bridge.

ity relationships in an image in terms of connectivity in higher-dimensional spaces, which I will call *attribute spaces* . As can be seen in Fig. 1, this leads to a more natural partitioning of the connected component into two squares and a single bridge. This effect is also shown in a practical application in Fig. 7.

This paper is organized as follows. First connected filters are described formally in Section 2, followed by second-order connectivities in Section 3. Problems with attribute filters using partitioning connectivities are dealt with in detail in this section. After this, attribute spaces are presented in section 4.

2. Connectivity and Connected Filters

As is common in mathematical morphology binary images X are subsets of some universal set E (usually $E = \mathbb{Z}^n$). Let $\mathcal{P}(E)$ be the set of all subsets of E. Connectivity in E can be defined using *connectivity classes* [10].

DEFINITION 1 *A connectivity class $\mathcal{C} \subseteq \mathcal{P}(E)$ is a set of sets with the following three properties:*

1 $\emptyset \in \mathcal{C}$

2 $\{x\} \in \mathcal{C}$

3 for each family $\{C_i\} \subset \mathcal{C}$, $\cap C_i \neq \emptyset$ *implies* $\cup C_i \in \mathcal{C}$.

This means that both the empty set and singleton sets are connected, and any union of connected sets which have a nonempty intersection is connected.

Any image X is composed of a number of connected components or *grains* $C_i \in \mathcal{C}$, with i from some index set I. For each C_i there is no set $C \supset C_i$ such that $C \subseteq X$ and $C \in \mathcal{C}$. If a set C is a grain of X we denote this as $C \lessdot X$.

An alternative way to define connectivity is through *connected openings*, sometimes referred to as connectivity openings [1].