Chapter 11

STRICT IMPLICATION, ENTAILMENT, AND MODAL ITERATION (1955)

Ever since C. I. Lewis offered the concept of “strict implication,” defined explicitly in terms of logical possibility \((p \rightarrow q \equiv_{df} \Diamond(p \cdot \sim q))\) and implicitly by the axioms of his system of strict implication, as corresponding to what is ordinarily meant by “deducibility” or “entailment,” there have been analytic philosophers who denied this correspondence. They denied it specifically because of the paradoxes of strict implication: that a necessary proposition is strictly implied by any proposition and an impossible proposition strictly implies any proposition. These theorems, it is maintained, do not hold for the logical relation ordinarily associated, both in science and in conversational language, with the word “entailment.” It is my aim in this paper to show that it is extremely difficult, if not downright hopeless, to maintain this distinction.

I shall refer specifically to a subtle paper by C. Lewy (Lewy 1950), which deals with the intriguing problem of modal iteration, and which emphatically endorses the distinction here to be scrutinized.

Let me begin by presenting a brief argument against the distinction which seems to me conclusive, though I do not intend to rest my case on it. It is simply that anybody who wishes to maintain the distinction must abandon one or the other of two propositions which seem equally unquestionable: (a) \(p\) is necessary if and only if \(\sim p\) is impossible, (b) \(p\) entails \(q\) if and only if it is necessary that \(p\), then \(q\)—where “if \(p\), then \(q\)” is a material implication.

For, from the conjunction of (a) and (b) we can deduce: \(p\) entails \(q\) if and only if \((p\ and \sim q)\) is impossible. Since the impossibility of the latter conjunction follows from the impossibility of \(p\), we have already the conclusion that an impossible proposition entails any proposition—which is what those who insist on the difference between entailment and strict implication deny. Notice that this argument does not presuppose that either (a) or (b) are definitions of “necessity” and “entailment” respectively. The conclusion follows even if the equivalences asserted by (a) and (b) are just material.

In the above mentioned essay, Lewy confesses that he cannot completely de-
fine “entailment,” but thinks that he is nevertheless justified in distinguishing entailment from strict implication because he can state two conditions which are necessary for $p$ to entail $q$, over and above the condition that $p$ strictly imply $q$, and which are not necessary for $p$ to strictly imply $q$: $p$ entails $q$ only if (1) “$R$ counts in favor of $p$” strictly implies “$R$ counts in favor of $q$” and (2) “$R$ counts against $q$” strictly implies “$R$ counts against $p$.” Lewy uses “counting in favor of” as a primitive concept; the illustrations he gives show that it is meant as a very broad concept which covers both “confirming evidence” in the sense of inductive logic and deductive entailment as special cases. Thus he would say presumably that a sample of black ravens counts in favor of “all ravens are black,” but also that the proposition “all birds are black” would, if it were true, count in favor of “all ravens are black.” He gives the following examples of strict implications which are not entailments because they either do not satisfy (1) or do not satisfy (2): “the proposition that there is nobody who is a brother and is not male is necessary” strictly implies “there is nobody who is a sister and is not female,” because the implied proposition is necessary and a necessary proposition is strictly implied by any proposition. But it is no entailment, he says, because (1) is not satisfied. (1) is not satisfied because it is logically possible that there should be an $R^1$ which counts in favor of the first proposition but is irrelevant to the truth of the second proposition. Lewy does not produce an example of such an $R$, but he might have produced the following: the concept being a brother is identical with the concept being a male sibling. Perhaps he would say that this proposition—the classical example of a “correct analysis” in Moore’s sense—entails, and therefore counts in favor of, the modal proposition “it is necessary that there are no brothers that are not male”; and surely we could agree that this proposition is irrelevant to the strictly implied proposition, since the latter does not contain the concept being a brother at all. His example of a strict implication which fails to satisfy condition (2) is, however, more convincing, since it involves nothing more problematic than that empirical evidence is, favorably or unfavorably, relevant only to contingent propositions: the false contingent proposition “Cambridge is larger than London” is strictly implied by the impossible proposition “there is somebody who is a brother and is not male,” but while there is empirical evidence counting against the implicite, there can be no empirical evidence that counts against the logically impossible implicans. If it were significant to say “since so far no brother has been found anywhere that was not male, it is unlikely that there is somebody who is a brother and is not male,” then the sentence “there is somebody who is a brother and is not male” would presumably express a contingent proposition.

Lewy, then, assumes the following principle: if $p$ entails $q$, then, for any $R$, “$R$ counts in favor of $p$” strictly implies “$R$ counts in favor of $q$”; and he be-

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1 I take Lewy’s “$R$” to be a propositional variable, such that “counting in favor of” designates, like “strict implication” and “entailment” in his usage, a relation between propositions.