MODEL REFERENCE CONTROL IN INVENTORY AND SUPPLY CHAIN MANAGEMENT

The implementation of a more suitable cost function

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Abstract: A method of model reference control is investigated in this study in order to present a more suitable method of controlling an inventory or a supply chain. The problem of difficult determining of the cost of change made in the control in supply chain related systems is studied and a solution presented. Both model predictive controller and a model reference controller are implemented in order to simulate results. Advantages of model reference control in supply chain related control are presented. Also a new way of implementing supply chain simulators is presented and used in the simulations.

1 INTRODUCTION

In recent years model predictive control (MPC) has gained a lot of attention in supply chain management and in inventory control. It has been found to be a suitable method to control business related systems and very promising results has been shown in many studies. The main idea in MPC has remained the same in most studies but many variations of the cost function can be found. Basically these cost functions, used in studies concerning MPC in supply chain management, can be separated in two different categories: quadratic and linear cost functions. The use of a linear cost function can be seen appropriate as it can take advantage of actual unit costs determined in the case. On the other hand these costs need to be fairly accurate to result as an effective control. Examples of studies using linear cost functions in supply chain control can be found in (Ydstie, Grossmann et al., 2003) and (Hennet, 2003). In this study we will no longer study the linear form of the cost function but concentrate on the quadratic form. The quadratic form of the cost function is used in, for example, (Tzafestas et al., 1997) and (Rivera et al., 2003). In supply chain management the question is not only about how to control the chain but also about what is being controlled. The traditional quadratic form of the cost function used in MPC has one difficulty when it comes to controlling an inventory or a supply chain. The quadratic form involves penalizing of changes in the controlled variable. Whether this variable is the order rate or the inventory level or some other actual variable in the business, it is always very difficult to determine the actual cost of making a change in this variable. In this study we present an effective way of controlling an inventory with MPC without the problem of determining the cost of changing the controlled variable. The method of model reference control will be demonstrated in inventory control and results presented. The structure of this paper is as follows. In Chapter 2 we will take a closer view on model predictive control and on the theory behind model reference control. In Chapter 3 we present simulations with both model predictive control and model reference control and do some comparisons between those two. Finally we conclude the results from our study in the last chapter, Chapter 4.

2 MODEL PREDICTIVE CONTROL

Model predictive control originated in the late seventies and has become more and more popular ever since. MPC itself is not an actual control strategy, but a very wide range of control methods which make use of a model of the process. MPC was originally developed for the use of process control but has diversified to a number of other areas of control, including supply chain management and

inventory control in which it has gained a lot of attention. Today MPC is the only modern control method to have gained success in real world applications. (Camacho and Bordons 2002), (Maciejowski 2002).

As stated earlier, Model Predictive Control is a set of control algorithms that use optimization to design an optimal control law by minimizing an objective function. The basic form of the objective function can be written as

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t + j | t) - w(t - j)]^2 + \sum_{j=1}^{N_u} \lambda(j)[\Delta u(t + j - 1)]^2.$$  

(1)

where $N_1$ and $N_2$ are the minimum and maximum cost horizons and $N_u$ is the control horizon. $\delta(j)$ and $\lambda(j)$ can be seen as the unit costs of the control, $w(t)$, $\hat{y}(t)$ and $\Delta u(t)$ are the reference trajectory, the predicted outputs and the change between current predicted control signal and previous predicted control signal, respectively. (Camacho and Bordons 2002).

The algorithm consists of two main elements, an optimization tool and a model of the system. The optimizer contains a cost function with constraints and receives a reference trajectory $w(t)$ to which it tries to lead the outputs as presented in Figure 1. The actual forecasting in MPC is done with the model which is used to predict future outputs $\hat{y}(t)$ on the basis of the previous inputs $u_P(t)$ and future inputs $u(t)$ the optimizer has solved as presented in Figure 1. These forecasts are then used to evaluate the control and a next optimization on the horizon is made. After all the control signals on the horizon are evaluated, only the first control signal is used in the process and the rest of the future control signals are rejected. This is done because on the next optimizing instant, the previous output from the process is already known and therefore a new, more accurate forecast can be made due to new information being available. This is the key point in the receding horizon technique as the prediction gets more accurate on every step of the horizon but also is the source of heavy computing in MPC. The receding horizon technique also allows the algorithm to handle long time delays. (Camacho and Bordons 2002).

### 2.1 Implementing the Cost Function

As presented in equation 1, the basic form of a MPC cost function penalizes changes made in control weighted with a certain parameter $\lambda$. This kind of damping is not very suitable for controlling an inventory or a supply chain due to the difficulty of determining the parameter $\lambda$. It usually is either the cost of change in inventory level or the cost of change in ordering. On the other hand the parameter $\lambda$ cannot be disregarded as it results as minimum-variance control which most definitely is not the control desired. Another problem with the basic form of MPC used in inventory control is the fact that it penalizes the changes made in ordering and not in inventory levels, which can cause unnecessary variations in the inventory level as will be shown later in this study.

In this study we present a more suitable way to form the cost function used in a model predictive controller. The problematic penalizing of changes in the control is replaced with a similar way to the one