ON MODELING AND CONTROL OF DISCRETE TIMED EVENT GRAPHS WITH MULTIPLIERS USING (MIN, +) ALGEBRA

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Abstract: Timed event graphs with multipliers, also called timed weighted marked graphs, constitute a subclass of Petri nets well adapted to model discrete event systems involving synchronization and saturation phenomena. Their dynamic behaviors can be modeled using a particular algebra of operators. A just in time control method of these graphs based on Residuation theory is proposed.

1 INTRODUCTION

Petri nets are widely used to model and analyze discrete-event systems. We consider in this paper timed event graphs\(^1\) with multipliers (TEGM’s). Such graphs are well adapted for modeling synchronization and saturation phenomena. The use of multipliers associated with arcs is natural to model a large number of systems, for example when the achievement of a specific task requires several units of a same resource, or when an assembly operation requires several units of a same part. Note that TEGM’s can not be easily transformed into (ordinary) TEG’s. It turns out that the proposed transformation methods suppose that graphs are strongly connected under particular server semantics hypothesis (single server in (Munier, 1993), or infinite server in (Nakamura and Silva, 1999)) and lead to a duplication of transitions and places.

This paper deals with just in time control, i.e., fire input transitions at the latest so that the firings of output transitions occur at the latest before the desired ones. In a production context, such a control input minimizes the work in process while satisfying the customer demand. To our knowledge, works on this tracking problem only concern timed event graphs without multipliers (Baccelli et al., 1992, §5.6), (Cohen et al., 1989), (Cottenceau et al., 2001).

TEGM’s can be handled in a particular algebraic structure, called dioid, in order to do analogies with conventional system theory. More precisely, we use an algebra of operators mainly inspired by (Cohen et al., 1998a), (Cohen et al., 1998b), and defined on a set of operators endowed with pointwise minimum operation as addition and composition operation as multiplication. The presence of multipliers in the graphs implies the presence of inferior integer parts in order to preserve integrity of discrete variables used in the models. Moreover, the resulting models are non linear which prevents from using a classical transfer approach to obtain the just in time control law of TEGM’s. As alternative, we propose a control method based on "backward" equations.

The paper is organized as follows. A description of TEGM’s by using recurrent equations is proposed in Section 2. An algebra of operators, inspired by (Cohen et al., 1998a), (Cohen et al., 1998b), is introduced in Section 3 to model these graphs by using a state representation. In addition to operators \(\gamma, \delta\) usually used to model discrete timed event graphs (without multipliers), we add the operator \(\mu\) to allow multipliers on arcs. The just in time control method of TEGM’s is proposed in Section 4 and is mainly based on Residuation theory (Blyth and Janowitz, 1972). After recalling basic elements of this theory, we recall the residuals of operators \(\gamma, \delta\), and give the residual of operator \(\mu\) which involves using the superior integer part. The just in time control is expressed as the great-

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\(^1\)Petri nets for which each place has exactly one upstream and one downstream transition.
est solution of a system of "backward" equations. We give a short example before concluding.

2 RECURRENT EQUATIONS OF TEGM’s

We assume the reader is familiar with the structure, firing rules, and basic properties of Petri nets, see (Murata, 1989) for details.

Consider a TEGM defined as a valued bipartite graph given by a five-tuple \((P, T, M, m, \tau)\) in which:
- \(P\) represents the finite set of places, \(T\) represents the finite set of transitions;
- a multiplier \(M\) is associated with each arc. Given \(q \in T\) and \(p \in P\), the multiplier \(M_{pq}\) (respectively, \(M_{qp}\)) specifies the weight (in \(\mathbb{N}\)) of the arc from transition \(q\) to place \(p\) (respectively, from place \(p\) to transition \(q\)) (a zero value for \(M\) codes an absence of arc);
- with each place is associated an initial marking \((m_p)\) assigns an initial number of tokens (in \(\mathbb{N}\)) in place \(p\).

The counter variables of a TEGM (under Assertion 1 that they are immediately available at time 0, that is, \(\{q' \in \ast q\}\) for details. We assume the reader is familiar with the structure, firing rules, and basic properties of Petri nets, see (Baccelli et al., 1992) for details.

Remark 1 We disregard without loss of generality firing times associated with transitions of a discrete event graph because they can always be transformed into holding times on places (Baccelli et al., 1992, §2.5).

Definition 1 (Counter variable) With each transition is associated a counter variable: \(x_q\) is an increasing map from \(\mathbb{Z}\) to \(\mathbb{Z} \cup \{\pm \infty\}\), \(t \mapsto x_q(t)\) which denotes the cumulated number of firings of transition \(q\) up to time \(t\).

Assertion 1 The counter variables of a TEGM (under the earliest firing rule) satisfy the following transition to transition equation:

\[ x_q(t) = \min_{p \in \ast q, q' \in \ast p} \left\lceil M^{-1}_{qp} (m_p + M_{pq} x_{q'}(t - \tau_p)) \right\rceil. \]  

(1)

Note the presence of inferior integer part to preserve integrity of Eq. (1). In general, a transition \(q\) may have several upstream transitions \(\{q' \in \ast q\}\) which implies that its associated counter variable is given by the min of transition to transition equations obtained for each upstream transition.

Example 1 The counter variable associated with transition \(q\) described in Fig. 1 satisfies the following equation:

\[ x_q(t) = \left\lceil a^{-1}(m + b x_{q'}(t - \tau)) \right\rceil. \]

Figure 1: A simple TEGM.

Example 2 Let us consider TEGM depicted in Fig. 2. The corresponding counter variables satisfy the following equations:

\[
\begin{align*}
x_1(t) &= \min(3 + x_3(t - 2), u(t)), \\
x_2(t) &= \min(\left\lceil \frac{2x_1(t-2)}{3} \right\rceil, \left\lceil \frac{6 + 2x_3(t-2)}{3} \right\rceil), \\
x_3(t) &= 3x_2(t - 1), \\
y(t) &= x_2(t).
\end{align*}
\]

Figure 2: A TEGM.

3 DIOID, OPERATORIAL REPRESENTATION

Let us briefly define dioid and algebraic tools needed to handle the dynamics of TEGM’s, see (Baccelli et al., 1992) for details.

Definition 2 (Dioid) A dioid \((D, \oplus, \otimes)\) is a semiring in which the addition \(\oplus\) is idempotent \((\forall a, a \oplus a = a)\). Neutral elements of \(\oplus\) and \(\otimes\) are denoted \(\varepsilon\) and \(e\) respectively.