1 Introduction

The central topic of this book is how to *describe the structures of probabilistic conditional independence* in a way that the corresponding mathematical model has both relevant interpretation and offers the possibility of computer implementation.

It is a mathematical monograph which found its motivation in artificial intelligence and statistics. In fact, these two fields are the main areas where the concept of conditional independence has been successfully applied. More specifically, graphical models of conditional independence structure are widely used in:

- the analysis of *contingency tables*, an area of discrete statistics dealing with categorical data;
- *multivariate analysis*, a branch of statistics investigating mutual relationships among continuous real-valued variables; and
- *probabilistic reasoning*, an area of artificial intelligence where decision-making under uncertainty is done on the basis of probabilistic models.

A (non-probabilistic) concept of conditional independence was also introduced and studied in several other calculi for dealing with knowledge and uncertainty in artificial intelligence (e.g. relational databases, possibility theory, Spohn's kappa-calculus, Dempster-Shafer's theory of evidence). Thus, the book has a multidisciplinary flavor. Nevertheless, it certainly falls within the scope of *informatics* or *theoretical cybernetics*, and the main emphasis is put on mathematical fundamentals.

The monograph uses concepts from several branches of mathematics, in particular measure theory, discrete mathematics, information theory and algebra. Occasional links to further areas of mathematics occur throughout the book, for example to probability theory, mathematical statistics, topology and mathematical logic.

1.1 Motivational thoughts

The following "methodological" considerations are meant to explain my motivation. In this section six general questions of interest are formulated which may arise in connection with any particular method for describing conditional independence structures. I think these questions should be answered in order to judge fairly and carefully the quality and suitability of every considered method.

To be more specific, one can assume a general situation, illustrated by Figure 1.1. One would like to describe *conditional independence structures* (in short, CI structures) induced by probability distributions from a given fixed class of distributions over a set of variables N. For example, we can consider the class of discrete measures over N (see p. 11), the class of regular Gaussian measures over N (see p. 30), the class of conditional Gaussian (CG) measures over N (see p. 66) or any specific parameterized class of distributions. In other words, a certain *distribution framework* is specified (see Section A.9.5). In probabilistic reasoning, every particular discrete probability measure over N represents "global" knowledge about a (random) system involving variables of N. That means it serves as a knowledge representative. Thus, one can take an even more general point of view and consider a general class of knowledge representatives within an (alternative) uncertainty calculus of artificial intelligence instead of the class of probability distributions (e.g. a class of possibilistic distributions over N, a class of relational databases over N etc.).

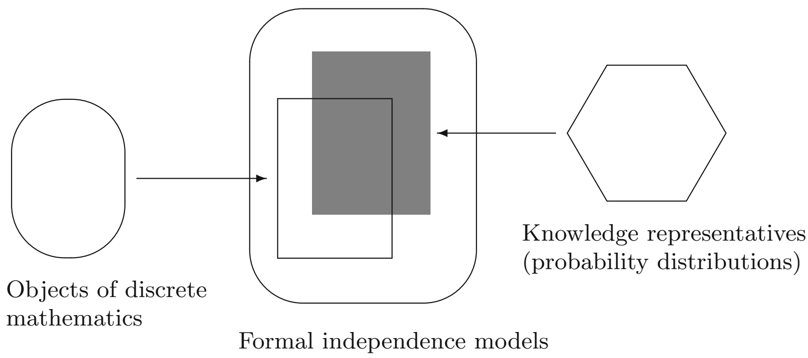

Fig. 1.1. Theoretical fundamentals (an informal illustration).

Every knowledge representative of this kind induces a formal independence model over N (for definition see p. 12). Thus, the class of induced conditional independence models is defined; in other words, the class of CI structures to be described is specified (the shaded area in Figure 1.1). One has in mind a