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Scalable Test Problems for Evolutionary Multiobjective Optimization

Kalyanmoy Deb, Lothar Thiele, Marco Laumanns and Eckart Zitzler

Summary. After adequately demonstrating the ability to solve different two-objective optimization problems, multiobjective evolutionary algorithms (MOEAs) must demonstrate their efficacy in handling problems having more than two objectives. In this study, we have suggested three different approaches for systematically designing test problems for this purpose. The simplicity of construction, scalability to any number of decision variables and objectives, knowledge of the shape and the location of the resulting Pareto-optimal front, and introduction of controlled difficulties in both converging to the true Pareto-optimal front and maintaining a widely distributed set of solutions are the main features of the suggested test problems. Because of the above features, they should be found useful in various research activities on MOEAs, such as testing the performance of a new MOEA, comparing different MOEAs, and better understanding of the working principles of MOEAs.

6.1 Introduction

Most earlier studies on multi-objective evolutionary algorithms (MOEAs) introduced test problems which were either too simple or not scalable in terms of number of objectives and decision variables. Some test problems were too complicated to visualize the exact shape and location of the resulting Pareto-optimal front. Schaffer's [1] study introduced two single-variable test problems (SCH1 and SCH2), which have been widely used as test problems. Kursawe's test problem [2], KUR, was scalable to any number of decision variables, but was not scalable in terms of the number of objectives. The same is true with Fonseca and Fleming's test problem [3], FON. Poloni et al.'s test problem [4], POL used only two decision variables. Although the mathematical formulation of the problem is non-linear, the resulting Pareto-optimal front corresponds to an almost linear relationship among decision variables. Viennet's test problem [5], VNT, has a discrete set of Pareto-optimal fronts, but was designed for three objectives only. Similar simplicity prevails in the existing constrained test problems [6, 7].

However, in 1999, the first author, for the first time, introduced a systematic procedure of designing two-objective test problems which are simple to construct and are scalable to the number of decision variables [8]. In these problems, the exact shape and location of the Pareto-optimal front are also known. The basic construction used two functionals, g and h^*, with non-overlapping sets of decision variables to introduce difficulties towards the convergence to the true Pareto-optimal front and to introduce difficulties along the Pareto-optimal front for an MOEA to find a widely distributed set of solutions, respectively. The construction procedure adopted in that study is not the only alternative for the test problem design and certainly many other principles are possible. In the absence of any other systematic construction procedure, those test problems have been used by many researchers since then. However, they have also been somewhat criticized for the relative independence feature of the functionals in achieving both the tasks. Such critics have overlooked an important aspect of that study. The non-overlapping property of the two key functionals in the test problems was introduced for ease of the construction procedure. That study also suggested the use of a procedure to map the original variable vector (say $\mathbf{y}$) on which an MOEA works to a different decision variable vector (say $\mathbf{x}$) with a transformation matrix: $\mathbf{x} = \mathcal{M}\mathbf{y}$. This way, although test problems are constructed for two non-overlapping sets from $\mathbf{x}$, each dependent variable x_i involves a correlation of all (or many) variables of $\mathbf{y}$. Such a mapping couples both aspects of convergence and maintenance of diversity and makes the problem harder to solve. However, Zitzler et al. [9] showed that six test problems designed based on an uncorrelated version of Deb's construction procedure were even difficult to solve exactly using the then-known state-of-the-art MOEAs.

In the recent past, many MOEAs have adequately demonstrated their ability to solve two-objective optimization problems by including three basic operators: (1) an elite-preserving operator, (2) a niche-preserving operator, and (3) a non-domination based selection operator. With the suggestion of a number of such MOEAs, it is time that they must be investigated for their ability to solve problems with more than two objectives. In order to help achieve such studies, it is therefore necessary to develop scalable test problems for a larger number of objectives. Besides testing an MOEA's ability to solve problems with a large number of objectives, the proposed test problems may also be used for systematically comparing two or more MOEAs. Since one such test problem can be used to test a particular aspect of multiobjective optimization, such as for convergence to the true Pareto-optimal front or maintenance of a good spread of solutions, etc., the test problems can be used to identify MOEAs which are better in terms of that particular aspect. For these reasons, these test problems may help provide a better understanding of the working principles of MOEAs, thereby allowing a user to develop better and more efficient MOEAs.

In the remainder of the study, we first describe the desired features needed in a test problem and then suggest three approaches for systematically