Variants of First-Order Modal Logics

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Abstract. In this paper we study proof procedures for some variants of first order modal logics, where domains may be either cumulative or freely varying and terms may be either rigid or non-rigid, local or non-local. We define both ground and free variable tableau methods, parametric with respect to the variants of the considered logics. The treatment of each variant is equally simple and is based on the annotation of functional symbols by natural numbers, conveying some semantical information on the worlds where they are meant to be interpreted.

1 Introduction

First order modal logics constitute a delicate and difficult subject (see for example [12] for an overview). In principle, their semantics could simply be obtained by extension of the modal propositional semantics: a first order modal structure can be built on the base of a set of first order classical interpretations (the “possible worlds”), connected by a binary relation (the accessibility relation). However, there are a number of different possibilities that can be considered, concerning for example:

The object domains: is there any relation between the universes of the different worlds? They can be required to be the same for all worlds (constant domains), they can bear no relation one to the other (varying domains), or the object domains can vary, but monotonically, i.e. if \( w' \) is accessible from \( w \), then the object domain of \( w \) is included in the domain of \( w' \) (cumulative domains).

The designation of terms: is the denotation of terms the same in all worlds or can it vary? When the answer is positive, then designation is taken to be rigid, otherwise it is non-rigid (often, mixed approaches are of interest, too, where the interpretations of some symbols are rigid, others are not).

The existence of objects: does the extension of any ground term belong to every object domain or not? If the answer is positive, then we assume terms to be local, otherwise terms are non-local.

So, several variants of quantified modal logics (QMLs) are possible, just by choosing different combinations of the cases considered above. We call them DDE variants (Domains/Designation/Existence variants) of QML. Obviously, the logic also depends on its propositional kernel, so that we have a four-dimensional...
space of possible QMLs. As often happens with modal logics, different choices are appropriate for different applications. A discussion can be found in [12] as well as [11].

Some DDE variants can easily be given a proof theoretic characterization, some are harder to treat. The easiest approach is obviously obtained just by adding the principles of classical logic to a propositional modal system. What we obtain then is a logic with cumulative domains, rigid designation and local terms. In fact, such a logic validates the converse of Barcan Formula (CBF), that characterizes cumulative domains: $\Box \forall x A \rightarrow \forall x \Box A$. Rigid designation and locality of terms are consequences of the instantiation rule of classical logic $\forall x A \rightarrow A[t/x]$. For example, $\Box p(c)$ follows from the instantiation rule and $p(c) \land \forall x (p(x) \rightarrow \Box p(x))$, but it is not a logical consequence of the latter formula alone, if the denotation of $c$ can vary from world to world (the example is from [16]).

Besides the axiomatic characterization, some first order modal logics with cumulative domains, rigid designation and local terms have been given sequent and tableau calculi [9,10,15], natural deduction [5], matrix proof procedures [19], resolution style calculi [1,7], and translation based procedures [3,18].

Now, while translation methods are general enough to treat also other DDE variants of QML (for instance, [3] deals with constant domains logics, where designation may be rigid as well as non rigid, and [18] handles all the DDE variants of QML), the direct methods are less flexible. For instance, it can be proved that constant domain logics cannot be treated by standard tableau systems (cut free sequent calculi). The addition of prefixes labelling tableau nodes solves this problem [9], as matrix methods do [19]: in both kinds of calculi in fact it is possible to analyse more than one possible world at a time, and this allows the proof to "go back and forth" (the same mechanism solves the problem of symmetric logics). In [9] all the variants of QML concerning the object domains of possible worlds are treated by prefix tableau methods and some suggestions on how to treat the varying domains case in calculi without prefixes can be found. A direct approach dealing with both varying domains and non-rigid symbols has a representative in [16], which defines a resolution method for epistemic logics, where terms can be annotated by a "bullet" constructor distinguishing rigid terms from non-rigid ones. A different direction is followed by [11], where the language of modal logic is enriched by means of a predicate abstraction operator, in order to capture differences on the denotation of terms, and a tableau proof procedure is presented for such a logic, with no restriction on the domains of possible worlds.

In general, direct proof methods can treat the less easy variants of QML either by modifications of classical and/or modal rules in an often cumbersome manner (if the axiomatic approach is taken), or else by attaching some kind of semantical information to the proof structures.

The aim of this work is to provide a treatment of some variants of QML, where the explicit intrusion of semantics in the proof structures is kept as simple and minimal as possible. We propose calculi in the tableau style, that allow us to treat all the DDE variants of QML (that are formally defined at the end of this section), with the exclusion of constant domain logics, that we believe