The Power of Mobility: Four Membranes Suffice

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Abstract. We continue the study of P systems with mobile membranes introduced in [6], which is a variant of P systems with active membranes having none of the features like polarizations, label change and division of non-elementary membranes. This variant was shown to be universal using only the simple operations of endocytosis and exocytosis; moreover, if elementary membrane division is allowed, it is capable of solving hard problems. Here, we investigate the power of the two operations (endocytosis, exocytosis) in more detail: 2 membranes can generate sets of vectors outside PsMAT, and four membranes give universality.

1 Introduction

P systems are a class of distributed parallel computing models inspired from the way the living cells process chemical compounds, energy, and information. One of the central operations in cell biology is cell division, and with this inspiration, P systems with active membranes were introduced in [7]. This variant was shown to be computationally universal as well as to be able to solve hard problems. The features used by this variant include the use of polarizations (+, -, 0) and division of non-elementary as well as elementary membranes, giving rise to an exponential workspace. These features are quite powerful, thus making the system powerful. Many attempts have been made to define equivalent systems having none of the above features, but in general, removal of one feature has requested the introduction of other powerful operations [8], [9]. [6] is an attempt in this direction, wherein we introduce a variant of P systems with none of the above mentioned features, but instead use two simple operations: endocytosis and exocytosis. These operations are different and simpler than the operations considered in [1], [2], [4] and [3].

In this paper, we take a closer look at the power of endocytosis and exocytosis rules. We have investigated the generative capacity of systems with 2 and 4 membranes and understand that endocytosis and exocytosis have a surprising power: universality is obtained with 4 membranes in contrast to achieving the same using label changing, membrane division and membrane dissolution keeping at most 3 membranes all the while [9].
2 Some Prerequisites

We refer to [5], [10] for the elements of formal language theory we use here. We only specify that for a string $x \in V^*$ and a symbol $a \in V$, we denote by $|x|$ the length of $x$ and by $|x|_a$ the number of occurrences of the symbol $a$ in the string $x$. For $w \in V^*$ with $V = \{a_1, \ldots, a_n\}$, we denote by $\Psi_V(w)$ the Parikh vector of $w$, that is, $\Psi_V(w) = (|w|_{a_1}, \ldots, |w|_{a_n})$; this is extended to languages in a natural way. For a family $FL$ of languages, we denote by $PsFL$ the family of Parikh sets of vectors associated with languages in $FL$.

A multi set over an alphabet $V$ is represented by a string over $V$ (and by all its permutations) and each string precisely identifies a multi set; the Parikh vector associated with the string indicates the multiplicities of each element of $V$ in the corresponding multi set. For basic elements of membrane computing we refer to [8]; for the state-of-the art of the domain, the reader may consult the bibliography from the web address http://psystems.disco.unimib.it.

We recall the definition of the family $MAT$. A context-free matrix grammar without appearance checking is a construct $G = (N, T, S, M)$ where $N, T$ are disjoint alphabets of non-terminals and terminals, $S \in N$ is the axiom, and $M$ is a finite set of matrices of the form $(A_1 \rightarrow x_1, \ldots, A_n \rightarrow x_n)$ of context-free rules. For a string $x$, a matrix $m : (r_1, \ldots, r_n)$ is executed by applying the productions $r_1, \ldots, r_n$ one after the another, following the order in which they appear in the matrix. We write $w \Rightarrow m z$ if there is a matrix $m : (A_1 \rightarrow x_1, \ldots, A_n \rightarrow x_n)$ in $M$ and the strings $w_1, \ldots, w_{n+1}$ in $(N \cup T)^*$ such that $w = w_1, w_{n+1} = z$, and for each $i = 1, 2, \ldots, n$ we have $w_i = w'_i A_i w''_i$, $w_{i+1} = w'_i x_i w''_i$. The language generated by $L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$. The family of languages generated by context-free matrix grammars is denoted by $MAT$.

For proving computational universality, we use the notion of a matrix grammar with appearance checking in the improved strong binary normal form, introduced in [6]. Such a grammar is a construct $G = (N, T, S, M, F)$, where $N = N_1 \cup N_2 \cup \{S, \#\}$, with these three sets mutually disjoint, two distinguished symbols $B^{(1)}, B^{(2)} \in N_2$, and the matrices in $M$ of one of the following forms:

1. $(S \rightarrow X A)$, with $X \in N_1, A \in N_2$,
2. $(X \rightarrow Y, A \rightarrow x)$, with $X, Y \in N_1, A \in N_2, x \in (N_2 \cup T)^*$,
3. $(X \rightarrow Y, B^{(j)} \rightarrow \#)$, with $X, Y \in N_1, j = 1, 2$,
4. $(X \rightarrow a, A \rightarrow x)$, with $X \in N_1, A \in N_2, x \in T^*$.

Moreover, there is only one matrix of type 1 and $F$ consists of all the rules $B^{(j)} \rightarrow \#$, $j = 1, 2$, appearing in matrices of type 3; $\#$ is a trap-symbol, once introduced it is never removed. (Clearly, a matrix of type 4 is used only once, in the last step of a derivation.)

3 P Systems with Mobile Membranes

We now briefly recall P systems with mobile membranes introduced in [6].

A P system with mobile membranes is a construct

$$\Pi = (V, H, \mu, w_1, \ldots, w_n, R),$$