Type-I Topological Logic $C^1_t$ and Approximate Reasoning*

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Abstract. We introduce the consistent topological structure and neighborhood structure into the logical framework for providing the logical foundation and logical normalization for the approximate reasoning. We present the concept of the formulae mass, the knowledge mass and the approximating knowledge closure of the knowledge library by means of topological closure. We obtain the fundamental framework of type-I topological logics. In this framework, we present the type-I topological algorithm of the simple approximate reasoning and multi-approximate reasoning. In the frameworks of type-I strong topological logics, we present the type-I topological algorithm of multidimensional approximate reasoning and multiple multidimensional approximate reasoning. We study the type-I completeness and type-I perfection of the knowledge library in the framework of topological logical frameworks. We construct the type-I knowledge universe and prove that the second class knowledge universe of type-I is coincident with the first class knowledge universe of type-I, therefore the type-I knowledge universe is stable. We construct a self-extensive type-I knowledge library and the type-I expert system. In this expert system, the new approximate knowledge acquired by the self-extensive type-I knowledge library $K^t$ will not beyond the type-I approximate knowledge closure, $(K_0)^{−}$, of the initial knowledge library $K_0$. Therefore, the precision of all new acquired approximate knowledge of this automatic reasoning system will be controlled well by the type-I approximate knowledge closure $(K_0)^{−}$ of the initial knowledge library $K_0$.

1 Introduction

Many logistician have done a lot of research work for providing the logical foundation and logical normalization of the approximate reasoning. Guo-Jun Wang constructed a fuzzy propositional logical system $L^*$ and gave the $\alpha-3I$ algorithm

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of the approximate reasoning in [1]. In [2], Wang studied the logical metric space based on their integrate semantic theory. The approximate reasoning model in the framework of the classic propositional logic is also studied in [3]. Mingsheng Ying gave a logic model of approximate reasoning in [4] and studied approximate reasoning based on the fuzzy matching in [5]. We constructed the framework of the stratified fuzzy propositional logic in [6] and discovered an important logical property of Mandanian algorithm for fuzzy reasoning in [7]. All the above work motivate the authors to study the approximate reasoning model in the framework of topological logic.

For the approximate reasoning

\[
\begin{align*}
A & \rightarrow B \\
A^\ast & \rightarrow B^\ast,
\end{align*}
\]

(1)

the MP rule in the classic propositional logic \( \mathbb{C} \) will be invalid, if \( A^\ast \neq A \). This case occurs frequently in the theory and application of the artificial intelligence, where the matching degree is applied to deal with such case. Considering the metric \( d(A^\ast, A) \), approximate degree \( q(A^\ast, A) \), similarity \( s(A^\ast, A) \), etc., the essential of all above definitions are measure of the approximation between the input \( A^\ast \) and the antecedent \( A \) of the knowledge \( A \rightarrow B \). Based on such approximation measures, the conclusion \( B^\ast \) of the approximate reasoning can be obtained so as to keep close to the descendant \( B \) of the knowledge \( A \rightarrow B \), according to the corresponding approximation measures. From the view of the abstract mathematics, the approximation of two objects can be described by topological structure. This thought lead up to the topics of topological logics and the topics of approximate reasoning in the topological logics. Such topological logical description of approximate reasoning may reflect the more essential relations between the approximate reasoning and the logics.

2 The Construction of Type-I Topological Logic \( \mathbb{C}^1_\mathcal{T} \)

Let \( \mathbb{C} = (\tilde{\mathbb{C}}, \tilde{\mathbb{C}}) \) be the classic propositional logic, \( \tilde{\mathbb{C}} \) is the syntax of \( \mathbb{C} \) and \( \tilde{\mathbb{C}} \) is the semantic of \( \mathbb{C} \). Let \( F(S) \) be the proposition set of \( \mathbb{C} \), which is also called formulae set. \( \rightarrow \) is the implication connective between the propositions. \( \mathcal{G}^\vdash \) is the set of all theorems in \( \tilde{\mathbb{C}} \) and \( \mathcal{G}^{\vdash} \) is the set of all tautologies in \( \tilde{\mathbb{C}} \). According to the soundness theorem and the completeness theorem for \( \mathbb{C} \), we have \( \mathcal{G}^\vdash = \mathcal{G}^{\vdash} \).

Suppose \( \mathcal{T} \) is a topology on the formulae set \( F(S) \), then \( (F(S), \mathcal{T}) \) is a topological space. For any formulae \( A, B, A \rightarrow B \in F(S) \), let \( \mathcal{U}(A), \mathcal{U}(B), \mathcal{U}(A \rightarrow B) \) denote their neighborhoods respectively under the topology \( \mathcal{T} \). Let \( "^-" \) denote the topological closure operator decided by \( \mathcal{T} \).

If for each formula \( A \rightarrow B \in F(S) \), we have

\[
\{A\}^- - \{B\}^- = \{A \rightarrow B\}^-
\]

(2)