Seeking Multiobjective Optimization in Uncertain, Dynamic Games

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Abstract. If the decisions of agents arise from the solution of general unconstrained problems, altruistic agents can implement effective problem transformations to promote convergence to attractors and draw these fixed points toward Pareto optimal points. In the literature, algorithms have been developed to compute optimal parameters for problem transformations in the seemingly more restrictive scenario of uncertain, quadratic games in which an agent's response is induced by one of a set of potential problems. This paper reviews these developments briefly and proposes a convergent algorithm that enables altruistic agents to relocate the attractor at a point at which all agents are better off, rather than optimizing a weighted function of the agents' objectives.

1 Introduction

Initially confined to mathematical modeling of decision making and economic players, game theory has drawn the attention of researchers and practitioners from a wide range of fields, including robotics, artificial intelligence, and control theory. The interest arises in part from the ever increasing complexity of systems composed of distributed, autonomous agents that may collaborate and compete for the resources in operating these systems. But also because the game-theoretic framework neatly generalizes problems in these domains, offering standard concepts of stability (*Nash equilibria*) and global optimality (*Pareto efficiency*).

In robotics, game theory has laid the foundation to generalize path planning problems to motion planning strategies [10,9]. While the former concerns the computation of a collision-free path from an initial configuration to a goal state, the latter explicitly considers dynamic environments [12], uncertainty in sensor data and motion, and constraints, among others. Within the game-theoretic framework, the motion planning strategy problem can be formulated to account for these complications in a unified way, whereby the uncertainties and decisions from other entities are viewed as competitive players from the part of the robot.

In multiagent systems, game theory has been the cornerstone of the extension of the single agent learning problem to optimal decision making in stochastic games [2]. In these generalized contexts, the optimal decision policy of one agent is dependent upon the policies of the others which creates policy dynamics as the

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agents pursue their own interests. Here the concept of Nash equilibrium comes into play to formalize the concept of optimality: the agents' policies can be optimal only if each agent has no incentive to deviate from its policy, which can be attained only at Nash equilibria. The existence of and convergence to Nash equilibria are recurring themes in the literature. These issues have been addressed recently in more practical, but complicated scenarios in which the agents have limited abilities and not necessarily make optimal decisions [4]. Further, promising policy-learning algorithms that promote convergence to Nash equilibria of stochastic games have also been the focus of research [3].

In control theory, the game-theoretic framework has been applied to model the operation of large, dynamic systems with networks of distributed control agents [11]. The distributed game can arise from (i) imperfections in a decomposition of the dynamic optimization problem of operating the system into a set of distributed, dynamic optimization sub-problems, one for each agent, or (ii) from a distributed specification of the agents' sub-problems that consider only the local state and local goals in operating the system. Here the policies are not given explicitly as in multiagent systems, nor focused on a single agent as in the robotics case, but rather specified as the solution of optimization problems. The top-down approach whereby the overall problem is decomposed, also known as distributed model predictive control, is somewhat mature [5,7] with conditions for perfect decomposition of constrained problems, convergence of distributed iterative processes to fixed points, and convergence to Pareto efficient solutions. On the other hand, the bottom-up approach, in which the distributed problems are not given by a central agency, is much less mature. Preceding research has produced means for altruistic agents, those that are interested in the overall performance and that can be programmed, to promote convergence to Nash equilibria and draw their location toward the Pareto optimal set [6]. The means are simple, yet effective problem transformations induced by factors referred to as *altruistic factors* for convergence and attractor location. In this paper, we extend these developments by proposing an algorithm that enables altruistic agents to move the attractor to locations that optimize not a combination, but rather optimize simultaneously an arbitrary subset of the agents' objectives in quadratic games. Besides being interesting in their own right, quadratic games can appear in decompositions of distributed model predictive control applied to linear systems and also locally approximate more general games.

2 Preliminaries

Definition 1. *$\mathcal{P}_m = \{P_m^1, \ldots, P_m^{\kappa_m}\}$ is the family of problems being solved by agent m which are given by:*

$$P_m^k : \underset{x_m}{\text{Minimize}}\ f_m^k(x_m, y_m)$$

where: $x_m \in \mathbb{R}^{n_m}$ is the agent's decision vector; y_m contains the decisions of the remaining agents; f_m^k is a smooth objective function; and κ_m is the cardinality of the agent's problem set.