1 Introduction

1.1 Introduction to Reduced Order Systems

A control engineer is fortunate to be able to work in a field where elegant mathematics often leads to a useful end result. Some of the most exciting examples of this aspect of control engineering have become widely known. For example Wiener filtering [1], has provided us a method of optimizing the design of constant coefficient linear filter to reduce the impact of noise. This work was thought to be so useful that it was classified during World War II and not published until 1948. In the sixties, Kalman filtering [2], provided a similar kind of a break through, but it was done in a state space setting. Both of these techniques were based on a knowledge of the spectral content of the disturbing signals, and both were aimed at estimation of signals in the presence of noise. Luenberger [3], developed a methodology for estimating unmeasured states and his method did not require any knowledge of stochastic processes. Luenberger's state estimates are referred to as observers. Luenberger observers and Kalman filter both provide a mechanism for using estimates of unmeasured states in a linear feedback controller. In both cases there is a separation theorem available for the design of the estimator and controller. In the case of stochastic models, the optimal stochastic control [4], makes use of a Kalman filter to provide state estimates. The combined controller and estimator makes up an intermediate dynamical system which could be thought of as a compensator. This famous result becomes known as the "L.Q.G." [5] result, meaning that it applied to linear systems with quadratic performance measures, and gaussian disturbances and initial conditions. Few would deny the mathematical elegance of this solved problem. Often the technique can lead to a useful end result. But the dimension of the Kalman filter used in the compensator could present practical difficulties with respect to implementation and this is still an issue thirty years later.

The principal difficulty is that the order of the Kalman filter is the same as the order of the original system. If one is dealing with a large scale sys-

A.A. Jalali, C.S. Sims, and P. Famouri: Reduced Order Systems, LNCIS 343, pp. 1–11, 2006.

tem, a system of high dimensionality such as a power system or a flexible structure, then it may not be reasonable to implement the full order Kalman filter. The optimal solution designed without regard for the difficulty of implementation could be unsatisfactory because the required calculations cannot be made in the time available. As an attending to sequential computation, the designer might choose an analog implementation or a massively parallel digital implementation of the filter in order to achieve reasonable through put [6]. The difficulty then might be hardware requirements or power requirements. To this author it appears that the viewpoint that there will be technological advances eliminating the need for concern regarding the dimensionality of a filter or controller, is probably naive. Though one might argue that technology moves faster than our ambition to solve complex systems problems, that doesn't seem to be the reality. In fact, technological break through motivate us to consider new and more challenging problems, for example, real time image processing, which might previously have been out of the question.

What has happened as a result of the above mentioned difficulty is that engineers, for a very long time now have been trying to reduce the dimensionality of controllers or filters. This model reduction may be done either explicitly or implicitly. It may be done either to simplify analysis or to simplify design or for both reasons. The approximation of infinite order, distributed systems by finite order lumped models is one case in point, which might be thought of as the ultimate reduced order problem. We are concerned here with explicit model reduction and the associated mathematics. There is rather a long history of background work on this topic, which has gone by different names over the years, but which has generally focused on simplifying dynamical structures.

1.2 Background

The primary point of view taken in this book is that of parameter optimization. That is, the structure of a filter or controller or smoother will be predetermined, and then the parameters that describe the structure will be optimized. The history of this approach is long. The book of Newton et.al. [7], represents some of the pioneering work in the optimization of fixed structure controllers. Like the work of Wiener (which optimized both structure and parameters), this body of work has a setting in the context of stationary stochastic processes. Once state space techniques had been well established, work in the area of limited complexity controllers was referred to as specific optimal control or fixed configuration control [8-10]. Similar