10 Robust Reduced Order Filtering

10.1 Introduction

In this chapter we shall address topics dealing with adding robustness to the design of reduced order filters. We will show how one can treat a broad range of noise or disturbance terms characterized as having bounded energy, rather than as having white noise characteristics. For such additive arbitrary disturbances, we show how to guarantee a bound on the ratio of the energy in the error to the energy in the disturbances. In [1], a rigorous derivation of the results contained here was developed. We will present here, a derivation based on game theory which more closely corresponds to the calculus of variations approach taken throughout this text, and follows the method suggested by Banavar and Speyer [2].

In terms of robustness with respect to plant variations, we present an approach which is radically different and is based on a state dependent noise concept [3]. Bernstein [4] originally suggested this as a methodology to enhance robustness, and this approach has been explored by others as well.

10.2 Full Order Filtering in an H Infinity Setting

Consider a system described by the linear dynamical model

$$\dot{x}(t) = A(t)x(t) + \theta(t)w(t) \tag{10.2.1}$$

and having a measurement model

$$m(t) = C(t)x(t) + v(t) \tag{10.2.2}$$

where $w(t)$ of covariance matrix $Q(t)$ in previous chapters, and was modeled as white noise, and similarly $v(t)$ was modeled as having covariance matrix $\hat{R}(t)$ and was also white. Here we shall depart from that de-

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scription and characterize $w(t)$ and $v(t)$ as simply bounded energy functions. We shall try to ensure that the maximum value of the ratio of the energy in the error to the energy in the disturbance processes is limited to a value, γ. The vector to be estimated is

$$z(t) = Lx(t) \tag{10.2.3}$$

and the estimate is to be of the form

$$\hat{z}(t) = L\hat{x}(t)\,. \tag{10.2.4}$$

We define the estimation errors as

$$e_x(t) = x(t) - \hat{x}(t) \tag{10.2.5}$$

and

$$e(t) = z(t) - \hat{z}(t)\,. \tag{10.2.6}$$

Our objective is to ensure that the ratio maximum

$$J = \frac{\int_{t_o}^{t_f} e^T(t)Ue(t)dt}{\int_{t_o}^{t_f} w^T(t)\hat{Q}^{-1}w(t)dt + \int_{t_o}^{t_f} v^T(t)\hat{R}^{-1}v(t)dt} < \gamma\,. \tag{10.2.7}$$

Following the procedure of Banavar and Speyer [1], we will approach this optimization as a game. It is convenient to consider the following problem in that regard

$$\underset{\hat{z}(t)}{\text{minimize}}\left\{\underset{v(t),w(t)}{\text{maximum}}\left\{-\frac{1}{2\hat{\gamma}}\int_{t_o}^{t_f}\left[w^T(t)\hat{Q}^{-1}w(t) + v^T(t)\hat{R}^{-1}v(t)\right]dt + \frac{1}{2}\int_{t_o}^{t_f} e^T(t)Ue(t)dt\right\}\right\} \tag{10.2.8}$$

We substitute into this performance measure the following relationships

$$v(t) = m(t) - Cx(t) \tag{10.2.9}$$

and

$$e(t) = Le_x(t) = L\left[x(t) - \hat{x}(t)\right]. \tag{10.2.10}$$

Then our performance measure may be written as

$$\underset{\hat{x}(t)}{\text{minimize}}\left\{\underset{m(t),w(t)}{\text{maximum}}\left\{-\frac{1}{2\hat{\gamma}}\left[\int_{t_o}^{t_f} w^T(t)\hat{Q}^{-1}w(t)dt + \right.\right.\right.$$