APPENDIX B

We consider each of the terms in 7.4.2 individually.

Term 1:

From (7.3.21), we have

$$\frac{d}{dt}\left[\int_0^t \varphi(t,s)\Omega_1 y(s)ds\right] = \Omega_1 y(t) + \Gamma_1 \int_0^t \varphi(t,s)\Omega_1 y(s)ds \tag{B,1}$$

Term 2:

$$\frac{d}{dt}\left[\int_0^t E\left\{z(t)\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds\right] = E\left\{z(t)\nu^T(t)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(t) \tag{B.2}$$

$$+\int_0^t E\left\{\frac{dz(t)}{dt}\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds \,.$$

Let us now consider the two terms on the right-hand side of (B.2) separately. Using (7.3.26), we have

$$E\left\{z(t)\nu^T(t)\right\} = -E\left\{z(t)e^T(t|t)\right\}\Gamma_2^T + E\left\{z(t)\nu^T(t)\right\}\Omega_2^T \tag{B.3}$$

$$= -E\left\{e(t|t)e^T(t|t)\right\}\Gamma_2^T - E\left\{\hat{z}(t|t)e^T(t|t)\right\}\Gamma_2^T = -P(t|t)\Gamma_2^T - E\left\{\hat{z}(t|t)e^T(t|t)\right\}\Gamma_2^T$$

where $P(t|t)$ is the filtered error covariance, i.e.

$$P(t|t) = E\left\{e(t|t)e^T(t|t)\right\} .$$

In obtaining (B.3) we have made use of the fact that $z(t)$ is uncorrelated with $\nu(t)$ and that $z(t) = \hat{z}(t|t + e(t|t)$. From (7.3.30),

$$E\left\{\hat{z}(t|t)e^T(t|t)\right\} = \int_0^t \varphi(t,s)\Omega_1 E\left\{y(s)e^T(t|t)\right\}ds + \int_0^t g(t,s)E\left\{\nu(s)e^T(t|t)\right\}ds \tag{B.4}$$

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The second term of (B.4) is zero because of the projection equation (7.3.29). Substituting (B.4) into (B.3) we obtain

$$E\left\{z(t)\nu^T(t)\right\} = -P(t|t)\Gamma_2^T - \int_0^t \varphi(t,s)\Omega_1 E\left\{y(s)e^T(t|t)\right\}ds\,. \tag{B.5}$$

We now consider the second term of (B.2). Using (7.3.16) and (7.3.26) we have

$$\begin{aligned}\int_0^t E\left\{\frac{dz(t)}{dt}\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds &= \Gamma_1\int_0^t E\left\{z(t)\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds \\ &+\int_0^t E\left\{\left[\Omega_1 y(t) - \Omega_1\nu(t) + LBw(t)\right]\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds \\ = \Gamma_1\int_0^t E\left\{z(t)\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds &- \int_0^t \Omega_1 E\left\{y(t)e^T(s|s)\right\}\Gamma_2^T\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu^T(s)ds\end{aligned} \tag{B.6}$$

where we have made use of the fact that $\nu(s)$ is uncorrelated with $w(t)$ and $\nu(t)$ for s < t and that $y(t)$ is uncorrelated with $\nu(t)$ for s < t. Substituting (B.5) and (B.6) into (B.2) yields

$$\begin{aligned}&\frac{d}{dt}\left[\int_0^t E\left\{z(t)\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds\right] \\ &= -P(t|t)\Gamma_2^T\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(t) - \int_0^t \varphi(t,s)\Omega_1 E\left\{y(s)e^T(t|t)\right\}\Gamma_2^T ds\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(t) \\ &+\Gamma_1\int_0^t E\left\{z(t)\nu^T(s)\right\}\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)dt - \int_0^t \Omega_1 E\left\{y(t)e^T(s|s)\right\}\Gamma_2^T\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)dt\end{aligned} \tag{B.7}$$

Term 3:

Using (7.3.21)

$$\begin{aligned}&\frac{d}{dt}\left[\int_0^t \varphi(t,s)\Omega_1 R\Omega_2^T\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(t)ds\right] \\ &= \Omega_1 R\Omega_2^T\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(t) + \Gamma_1\int_0^t \varphi(t,s)\Omega_1 R\Omega_2^T\left(\Omega_2 R\Omega_2^T\right)^{-1}\nu(s)ds\end{aligned} \tag{B.8}$$