2 Static Problems

2.1 Introduction

The static problem can only provide a limited amount of insight regarding the ideas behind the study of reduced order systems. The reason is that system order inherently has to do with how the state of a system evolves in time, i.e. with the order of the differential or difference equations considered. Some insight into how one proceeds, however, can be gained by looking at the static case, and one can get some idea of the difficulties involved. We shall begin by looking at a static control problem.

2.2 Control with Perfect Information

In the problem considered, the error, e_c, represents a deviation away from a desired location of a vector being controlled. The vector, e_c, is linearly related to a larger vector of errors, e_f which is described by

$$e_f = Ax + Bu + w \tag{2.2.1}$$

where A and B are known matrices of dimension $n \times n$ and $n \times r$ respectively, x is a vector of perfectly measured variables, u is a vector of controls, and w is a vector of zero mean random disturbances. The vectors e_c and e_f are related by the equation

$$e_c = Le_f \tag{2.2.2}$$

where L is a known matrix, and is generally used to select the subset of the full order error vector, e_f, which is of interest. The vector e_c is therefore of reduced order relative to e_f since $\ell \le n$.

From Eqs. (2.2.1) and (2.2.2) we may write

$$e_c = LAx + LBu + Lw \tag{2.2.3}$$

A.A. Jalali, C.S. Sims, and P. Famouri: Reduced Order Systems, LNCIS 343, pp. 13–25, 2006.

or

$$e_c = q + B_1 u + w_1 \tag{2.2.4}$$

where

$$q = LAx \equiv \varphi x$$

$$B_1 = LB \text{ and } w_1 = Lw \tag{2.2.5}$$

Our objective is to make e_c small without using values of control which are too large.

Towards this end we will try to pick u to minimize the quadratic performance measure

$$J = E\left\{e_c^T Q e_c + u^T R u \middle| x\right\} \tag{2.2.6}$$

The matrices Q and R are respectively positive semi definite and positive definite symmetric weighting matrices. Their elements are chosen to reflect the relative importance of error and control effort, so that their elements are trade-off parameters. The expectation operator is required of course since the exact value of e_c is random. Substitution from Eq. (2.2.4) gives

$$J = E\left\{(q + B_1 u + w_1)^T Q (q + B_1 u + w_1) + u^T R u \middle| x\right\} \tag{2.2.7}$$

Setting $\frac{\partial J}{\partial u} = 0$ gives the equation for the optimal control

$$u = K_c q = K_c \varphi x \tag{2.2.8}$$

where

$$K_c = -\left[B_1^T Q B_1 + R\right]^{-1} B_1^T Q \tag{2.2.9}$$

Whenever $\ell \le n, K_c$ has fewer elements than it would otherwise have.