5 Smoothing

5.1 Introduction

Up to this point we have not considered reduced order smoothing problems, where data over an entire interval may affect an estimate at any time during the interval. Such problems have the characteristics that they have non causal solutions and so may not be implemented in real time. Since this is the case, the reader may wonder why we would be interested in a reduced order sub optimal solution, as obtained in [1] instead of a full order optimal solution as presented in [2] and [3]. The answer is simply that complexity of the solution is still a factor, even when the signal processing is done off-line. If one has a state model of very high order, one does not want to be required to have a smoother of corresponding high order due to the high complexity of such a solution. The nicest situation one can have is when both the processing equations and design equations are of limited complexity. It should be noted, however, that there is a difference between the two categories even for off line processing, because the Riccati (design) equation is solved only once, but the smoothing (processing) equations could be used repeatedly on vast amounts of data.

In this chapter, we will consider only the case of reduced order fixed interval smoothing. The type of solution that we get is referred to as a two-filter or Mayne-Fraser type of smoother [2], [3]. In later chapters we will return to the smoothing problem and treat problems having random boundary conditions at both initial and terminal time. We shall also look at different forms of the solution to the smoothing problem, and at different approaches for finding the solution.

5.2 The Reduced Order Smoothing Problem

We will assume that the dynamical model is as indicated by equation (4.2.1), and the observation model is as given by equation (4.2.2) where

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observations are made in the time interval, $t_0 \le t \le T$. The process and measurement noise is zero mean white noise as characterized by equation (4.2.3). The initial mean and variance are

$$\mathrm{Var}\{x(t_0)\} = E\{[x(t_0) - \mu_x(t_0)][x(t_0) - \mu_x(t_0)]^T\} = P_{xx}(t_0) - \mu_x(t_0)\mu_x^T(t_0) \qquad (5.2.1)$$

and the initial statistics and noise are independent. We will consider the fixed interval smoothing problem of estimating $z(t) = Lx(t)$ for every t in the interval $[t_0, T]$, given data over the entire interval. We will consider designing two-filters: a forward filter providing estimate $\hat{z}_f(t)$, and a backward filter providing estimate $\hat{z}_b(t)$. These two estimates are combined to provide the overall smoothed estimates

$$\hat{z}_s(t) = \alpha(t)\hat{z}_f(t) + \beta(t)\hat{z}_b(t)\,. \qquad (5.2.2)$$

The forward filter use the data from t_0 to t, as well as the a priori statistics as indicated in (5.2.1). Thus using (4.2.22) and (4.2.23), and considering a quadratic performance measure such as (4.2.14)

$$\dot{\hat{z}}_f(t) = [\sigma(t) - P_f(t)\gamma(t)]\hat{z}_f(t) + [\omega(t) - P_f(t)\rho(t)]m(t) \qquad (5.2.3)$$

where

$$\hat{z}_f(t_0) = L\mu_x(t_0) \qquad (5.2.4)$$

and

$$\dot{P}_f(t) = \sigma(t)P_f(t) + P_f(t)\sigma^T - P_f(t)\gamma(t)P_f(t) + \Xi(t) \qquad (5.2.5)$$

and

$$\dot{P}_f(t_0) = L(t_0)[P_{xx}(t_0) - \mu_x(t_0)\mu_x^T]L^T(t_0)\,. \qquad (5.2.6)$$

What remains is the design of the backwards filter, and the optimal choice of $\alpha(t)$ and $\beta(t)$.

5.3 The Backward Filter

The backward filter makes use of the information in $[t, T]$ and propagates in the opposite direction to the forward filter. Since this is so we consider the system dynamics described in terms of

$$\tau = T - t$$