9 Reduced Order Filtering for Flexible Space Structures

9.1 Introduction

Flexible space structures are often modeled by a large set of second order differential equations. A Kalman filter designed for such a model might not be a very practical idea because of the high dimensionality required and the associated complexity of implementation. Here we derive a class of reduced order filters which reduce the complexity of the design and filtering process. As an added feature, the reduced order filter performance is shown to be completely insensitive to system parameters for an important class of problems.

If a flexible space structure is modeled by a large system of ordinary Differential equations, it might be difficult to implement the associated Kalman filter because of its high dimensionality. The option of simply reducing the order of the model by discarding a number of modes from consideration might lead to a faulty description of the physical process, and a corresponding inaccuracy of the filter. The approach that we take here is closer to that of Bernstein et.at. [1], except that we can optimize over a finite time interval and can therefore consider non-stationary problems. Also, we enforce an observer constraint [2], as in [3,4], which simplifies the design procedure as well as the processing algorithms but may limit performance relative to that obtained using the projection equations [1]. The performance loss associated with requiring that the reduced order filter have an observer structure has been recently investigated [5].

In the case of problems considered here, we include the possibility of accelerometer measurements, a fact which leads to correlated process and measurement noise, and so a different algorithm than that presented in Section (4.2). Application of the procedure to a particular class of important examples leads to a class of filters whose design depends only on a matrix of signal to noise parameters, and is insensitive to other system parameters.

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9.2 The Mathematical Model

The dynamics of the system in modal coordinates is described by the equation

$$\ddot{\eta} + D_d\dot{\eta} + \Omega_n\eta = \hat{B}u + \hat{G}w \tag{9.2.1}$$

where $\eta, \dot{\eta}$ and $\ddot{\eta}$ are vectors of positions, velocities, and accelerations respectively. The vector w describes random process noise, while u might represent a set of control inputs.
The matrices D_d and Ω_n are of the form

$$D_d = \text{dia}\left(2\zeta_i w_i\right) \tag{9.2.2}$$

$$\Omega_n = \text{dia}\, w_i^2 \tag{9.2.3}$$

while $\hat{B}$ and $\hat{G}$ represent the matrices operating on the control vector and noise respectively. We assume that three types of measurements may be available

$$\begin{aligned} y_1 &= c_1\ddot{\eta} + v_1 \\ y_2 &= c_2\dot{\eta} + v_2 \\ y_3 &= c_3\eta + v_3 \end{aligned} \tag{9.2.4}$$

The process noise, $w(t)$, and measurement noise, $v_i(t)$ are assumed to be zero mean white noise, with correlation matrices

$$\begin{aligned} E\left\{w(t)w^T(\tau)\right\} &= Q\delta(t-\tau) \\ E\left\{v_i(t)v_i^T(\tau)\right\} &= R_i\delta(t-\tau) \end{aligned} \tag{9.2.5}$$

The noise terms are uncorrelated with each other, or with initial conditions.
We may define the state vector as comprised of positions and velocities

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} \tag{9.2.6}$$

Then, in the usual state space notation we have form (9.2.1)

$$\dot{x}(t) = Ax(t) + Bu(t) + \theta w(t)\,, \tag{9.2.7}$$

The same form as equation (3.2.1), where A is partitioned as