Reducing Right-Hand Sides for Termination

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Abstract. We propose two transformations on term rewrite systems (TRSs) based on reducing right-hand sides: one related to the transformation order and a variant of dummy elimination. Under mild conditions we prove that the transformed system is terminating if and only if the original one is terminating. Both transformations are very easy to implement, and make it much easier to prove termination of some TRSs automatically.

Preface

Before introducing the technical contents of this paper first I want to spend some personal words. Several years ago, around 1990, I was looking for a new research area. At that time I was employed at Utrecht University, and among other things I was responsible for a seminar in algebraic specification. Only vaguely I was aware of the area of term rewriting providing a way for implementation of algebraic specifications. Just before that a nice booklet appeared, in Dutch, about term rewriting. I liked this booklet, and decided to use it for my seminar. This booklet appeared to be the course material of a course by Jan Willem Klop at the Free University in Amsterdam, only 40 kilometers from Utrecht. I heard that the group around Jan Willem Klop was active in research in term rewriting, and that they had meetings every two or three weeks around this research, called TeReSe: term rewriting seminar. Since I liked the topic as I learned it from the booklet, and still was looking for a new research area, I decided to follow these meetings. There I met Jan Willem Klop and the people of his group: Aart Middeldorp, Fer-Jan de Vries, Roel de Vrijer, Vincent van Oostrom and Femke van Raamsdonk. I liked the meetings and the pleasant atmosphere, and very naturally and smoothly inside this area I found challenges that happened to grow out to my own research topics.

Now fifteen years have been passed, and I may look back to (co-)authoring dozens of papers related to this area. Although I have never been a member of Jan Willem's group, I realize that in the way sketched above Jan Willem and his group have played a crucial role in my development as a scientist. I am very grateful for this. To mark one issue, on several places in the present paper the underlying theory is based on completions of diagrams as you may see from the pictures if you browse through the paper. For sure this way of completion of diagrams, preferably in the setting of abstract reduction systems, is inspired by the way Jan Willem propagated to do so in these TeReSe meetings long ago.

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1 Introduction

Developing techniques for proving termination of TRSs is a challenging research area already for a long time. In recent years the emphasis in this area has shifted towards implementation: for new techniques to prove termination it is no longer sufficient that they can be used to prove termination of particular TRSs in theory, but also tools should be able to use these techniques to prove termination fully automatically. Several tools have been developed for this goal, and there is a yearly competition in which all of these tools are applied to an extensive set of examples (TPDB [20], the termination problem data base), and compared, see

`http://www.lri.fr/~marche/termination-competition/`.

In this paper we present two transformations on TRSs for which termination of the original TRS can be concluded from termination of the transformed TRS. Since these transformations are very easy to implement and proving termination of the transformed TRS by standard techniques is often much simpler than proving termination of the original TRS, they are very suitable to be used as preprocessing steps before using any of the tools.

Both transformations do not change left-hand sides, and reduce right-hand sides. In the first transformation, related to the *transformation ordering* [3] this is done by rewriting right hand sides using the same TRS. So here it is assumed that at least one right-hand side of a rule is not in normal form. In the second transformation, a variant of *dummy elimination* [8], the right-hand sides are decomposed with respect to a special symbol (a *dummy symbol*) that occurs in a right-hand side but in no left-hand side.

The technique of rewriting right-hand sides was considered before in [12], but there it was required that the whole TRS is non-overlapping (or a mild weakening of it), while our approach does not have such global restrictions. Our approach is based on the transformation ordering from [3] presented in a more abstract setting in [24]. The first approach to implement this was described in [17]. In order to use this technique for rewriting right-hand sides we had to adjust the underlying theory. In this paper all required theory is included.

For our present variant of dummy elimination the main theorem states that the original TRS is terminating if the transformed TRS is terminating, just as in [8]. However, in case of left-linearity we also have the converse, as we prove in this paper. Therefore the new variant is called *complete dummy elimination*, and is often stronger than the earlier version from [8].

For string rewriting the techniques described in this paper have been implemented in TORPA: Termination of Rewriting Proved Automatically [21], a tool developed by the author, which was the winner in the above mentioned competition in the category of string rewriting, both in 2004 and 2005.

For term rewriting the techniques described in this paper have been implemented in the tool TPA: Termination Proved Automatically, written by Adam Koprowski, [15]. In the above mentioned termination competition in 2005 this tool was third among 6 participants in the category of term rewriting, after AProVE [6] and TTT [13].