Model Theory for Process Algebra

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Abstract. We present a first-order extension of the algebraic theory about processes known as ACP and its main models. Useful predicates on processes, such as deadlock freedom and determinism, can be added to this theory through first-order definitional extensions. Model theory is used to analyse the discrepancies between identity in the models of the first-order extension of ACP and bisimilarity of the transition systems extracted from these models, and also the discrepancies between deadlock freedom in the models of a suitable first-order definitional extension of this theory and deadlock freedom of the transition systems extracted from these models. First-order definitions are material to the formalization of an interpretation of one theory about processes in another. We give a comprehensive example of such an interpretation too.

1 Introduction

Model theory is for some time now a very active branch of mathematical logic. Therefore, it looks to be worthwhile to introduce various techniques from model theory into the field of process algebra. This forms the greater part of our motivation to take up the work presented in this paper. With great pleasure, we contribute this paper to the Liber Amicorum in honor of the 60th birthday of Jan Willem Klop.

Usually, theories about processes such as ACP [1, 2] and CCS [3, 4] are equationally axiomatized. However, it is also possible to give first-order theories. An important advantage of a first-order approach is that it makes available the tool of first-order definition of predicates and operations on processes.

In this paper, we present a first-order extension of ACP and its main models. The first-order extension concerned includes a binary reachability predicate on processes with an associated first-order axiom schema for subprocess induction. The reachability predicate can be used to give first-order definitions of many general properties of processes, such as deadlock freedom and determinism, and the axiom schema for subprocess induction can then be used to verify whether

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processes have these properties. This is one of the interesting applications of first-order definitions of predicates on processes.

First-order definitions of predicates and operations on processes are generally indispensable for the formalization of an interpretation of one theory about processes in another. For example, a first-order definition of the deadlock freedom predicate permits the formalization of the interpretation of BPA in BPA_δ [2] (both are subtheories of ACP). By first-order definitions of operations on processes, we are able to formalize more complicated interpretations, such as the interpretation of BPPA [5, 6] in the first-order extension of ACP. If one theory is interpretable in another theory, then a model of the former theory can be obtained from each model of the latter theory by taking a submodel of a restriction of an expansion by definitions. The expansion concerns the first-order definable operations on processes needed in the formalization of the interpretation concerned; and the first-order definable predicate on processes needed in the formalization of the interpretation determines the domain of the submodel. This technique to construct models can be regarded as a first-order generalization of the SRM-technique from [7].

In this paper, we analyse the discrepancies between identity in the models of the first-order extension of ACP and external bisimilarity, i.e. bisimilarity of the transition systems extracted from these models. Besides external bisimilarity, we pay attention to observational equivalence; and we have a look at other related issues such as bisimilarity based on structural operational semantics and modal characterization of external bisimilarity. We also analyse the discrepancies between deadlock freedom in the models of a suitable first-order definitional extension of the first-order extension of ACP and external deadlock freedom, i.e. deadlock freedom of the transition systems extracted from these models. Additionally, we briefly consider the comparable discrepancies for determinism.

It happens that the first-order extension of BPA_δ, which is a subtheory of the first-order extension of ACP, gets great expressive power in case it is extended with restricted reachability predicates. Even the first-order extension of ACP can be interpreted in it. In this paper, we formalize the interpretation concerned. Thus, we provide a comprehensive example of the formalization of an interpretation of one theory about processes in another.

The structure of this paper is as follows. First of all, we introduce $\mathrm{BPA}_\delta^{\mathrm{fo}}$, the (finitary) first-order extension of an important subtheory of ACP, to wit BPA_δ (Sect. 2). Next, we consider some useful infinitary and second-order axioms (Sect. 3). After that, we introduce transition systems, bisimilarity of transition systems (Sect. 4) and full bisimulation models, the main models of $\mathrm{BPA}_\delta^{\mathrm{fo}}$ (Sect. 5). Thereupon, we analyse the discrepancies between external bisimilarity and identity in models of $\mathrm{BPA}_\delta^{\mathrm{fo}}$ (Sect. 6) and investigate the related external equivalence known as observational equivalence (Sect. 7). Following this, we have a closer look at bisimilarity based on structural operational semantics (Sect. 8) and the modal characterization of external bisimilarity (Sect. 9). Then, we extend $\mathrm{BPA}_\delta^{\mathrm{fo}}$ with a deadlock freedom predicate and analyse the discrepancies between external deadlock freedom and internal deadlock freedom in models