Network Game with Attacker and Protector Entities

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Abstract. Consider an information network with harmful procedures called attackers (e.g., viruses); each attacker uses a probability distribution to choose a node of the network to damage. Opponent to the attackers is the system protector scanning and cleaning from attackers some part of the network (e.g., an edge or a path), which it chooses independently using another probability distribution. Each attacker wishes to maximize the probability of escaping its cleaning by the system protector; towards a conflicting objective, the system protector aims at maximizing the expected number of cleaned attackers.

We model this network scenario as a non-cooperative strategic game on graphs. We focus on the special case where the protector chooses a single edge. We are interested in the associated Nash equilibria, where no network entity can unilaterally improve its local objective. We obtain the following results:

\begin{itemize}
  \item No instance of the game possesses a pure Nash equilibrium.
  \item Every mixed Nash equilibrium enjoys a graph-theoretic structure, which enables a (typically exponential) algorithm to compute it.
  \item We coin a natural subclass of mixed Nash equilibria, which we call matching Nash equilibria, for this game on graphs. Matching Nash equilibria are defined using structural parameters of graphs, such as independent sets and matchings.
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  \item We derive a characterization of graphs possessing matching Nash equilibria. The characterization enables a linear time algorithm to compute a matching Nash equilibrium on any such graph with a given independent set and vertex cover.
  \item Bipartite graphs are shown to satisfy the characterization. So, using a polynomial-time algorithm to compute a perfect matching in a bipartite graph, we obtain, as our main result, an efficient graph-theoretic algorithm to compute a matching Nash equilibrium on any instance of the game with a bipartite graph.
\end{itemize}
1 Introduction

Motivation and Framework. Consider an information network represented by an undirected graph. The nodes of the network are insecure and vulnerable to infection. A system protector (e.g., antivirus software) is available in the system; however, its capabilities are limited. The system protector can guarantee safety only to a small part of the network, such as a path or even a single edge, which it may choose using a probability distribution. A collection of attackers (e.g., viruses or Trojan horses) are also present in the network. Each attacker chooses (via a separate probability distribution) a node of the network; the node is harmed unless it is covered by the system protector. Apparently, the attackers and the system protector have conflicting objectives. The system protector seeks to protect the network as much as possible, while the attackers wish to avoid being caught by the network protector so that they be able to damage the network. Thus, the system protector seeks to maximize the expected number of attackers it catches, while each attacker seeks to maximize the probability it escapes from the system protector.

Naturally, we model this scenario as a strategic game with two kinds of players: the vertex players representing the attackers, and the edge player representing the system protector. The Individual Cost of each player is the quantity to be maximized by the corresponding entity. We are interested in the Nash equilibria associated with this game, where no player can unilaterally improve its Individual Cost by switching to a more advantageous probability distribution. We focus on the simplest case where the edge player chooses a single edge.

Summary of Results. Our results are summarized as follows:

- We prove that no instance of the game has a pure Nash equilibrium (pure NE) (Theorem 1).
- We then proceed to study mixed Nash equilibria (mixed NE). We provide a graph-theoretic characterization of mixed NE (Theorem 2). Roughly speaking, the characterization yields that the support of the edge player and the vertex players are an edge cover and a vertex cover of the graph and an induced subgraph of the graph, respectively. Given the supports, the characterization provides a system of equalities and inequalities to be satisfied by the probabilities of the players. Unfortunately, this characterization only implies an exponential time algorithm for the general case.
- We introduce matching Nash equilibria, which are a natural subclass of mixed Nash equilibria with a graph-theoretic definition (Definition 1). Roughly speaking, the supports of vertex players in a matching Nash equilibrium form together an independent set of the graph, while each vertex in the supports of the vertex players is incident to only one edge from the support of the edge player.
- We provide a characterization of graphs admitting a matching Nash equilibrium (Theorem 3). We prove that a matching Nash equilibrium can be computed in linear time for any graph satisfying the characterization once a suitable independent set is given for the graph.