Dynamic Routing Schemes for General Graphs
(Extended Abstract)

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Abstract. This paper studies approximate distributed routing schemes on dynamic communication networks. The paper focuses on dynamic weighted general graphs where the vertices of the graph are fixed but the weights of the edges may change. Our main contribution concerns bounding the cost of adapting to dynamic changes. The update efficiency of a routing scheme is measured by the number of messages that need to be sent, following a weight change, in order to update the scheme. Our results indicate that the graph theoretic parameter governing the amortized message complexity of these updates is the local density $D$ of the underlying graph, and specifically, this complexity is $\tilde{\Theta}(D)$. The paper also establishes upper and lower bounds on the size of the databases required by the scheme at each site.

1 Introduction

Motivation: The basic function of a communication network, namely, message delivery, is performed by its routing scheme. Subsequently, the performance of the network as a whole may be dominated by the quality of the routing scheme. Thus, constructing an efficient routing scheme is one of the most important tasks when dealing with communication network design.

We distinguish between static and dynamic routing schemes. In a static routing scheme the databases of the processors are tailored to the particular network topology. However, in most communication networks, the typical setting is highly dynamic, namely, even when the physical infrastructure is relatively stable, the network traffic load patterns undergo repeated changes. Therefore, for a routing scheme to be useful in practice, it should be capable of reflecting up-to-date load information in a dynamic setting, which may require occasional updates to the databases. Ideally, upon a topological change, only a limited number of messages are sent in order to update the databases. We rank the update efficiency of dynamic schemes by their message complexity, i.e., the amortized number of messages sent per topological change. Note that the message complexity also

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bounds from above the amortized number of graph vertices whose database needs to be modified per update operation, hence lower message complexity implies also lower accounting efforts and fewer interruptions to the vertices.

The efficiency of a dynamic scheme is measured not only by its message complexity but also by the quality of the routes it provides and by the memory complexities associated with it. Route quality is measured by the stretch factor of the scheme, i.e., the maximum ratio, over all pairs of nodes in the network, between the length of the route provided for them by the routing scheme, and the actual (weighted) distance between them in the network. We focus on \( \beta \)-approximate routing schemes, namely, ones that produce a route whose weighted length is perhaps not the shortest possible, but approximates it by a factor of at most \( \beta \), for some constant \( \beta > 1 \).

Another consideration is the amount of information stored at each vertex. We distinguish between the internal database \( \text{Data}(v) \) used by each node \( v \) to deduce the required information in response to online queries, and the additional external storage \( \text{Memory}(v) \) at each node \( v \), used during (offline) updates and maintenance operations. For certain applications, the internal database \( \text{Data}(v) \) is often kept in the router itself, whereas the additional storage \( \text{Memory}(v) \) is kept on some external storage device. Subsequently, the size of \( \text{Data}(v) \) is a more critical consideration than the total amount of storage needed for the information maintenance.

The current paper investigates schemes on dynamic settings involving changing link weights. The model studied considers a network whose underlying topology is a fixed graph, i.e., the vertices and edges of the network are fixed but the (positive integer) weights of the edges may change. At each time the weight of one of the edges can increase or decrease by a fixed quanta (which for notational convenience is set to be 1), as long as the weight remains a positive integer. (Our algorithms and bounds apply also for larger weight changes, as clearly, a weight change of \( \Delta > 1 \) can be handled, albeit naively, by simulating it as \( \Delta \) individual weight changes of 1. As our focus is on establishing the complexity bounds for the problem, no attempt was made to optimize the performance of our algorithms in case of large weight changes.)

This paper introduces dynamic \( \beta \)-approximate routing schemes that are efficient in terms of their message complexity. We also give lower bounds regarding the complexities of dynamic routing schemes. Our results may indicate that the graph theoretic parameter governing the message complexity is the local density of the underlying graph, defined as follows. For a graph \( G \) and integer \( r \geq 1 \), let \( N(v, r) \) denote the set of vertices at distance at most \( r \) from the node \( v \). Then the local density of \( G \) is \( D = \max_{v, r} \{|N(v, r)|/2r\} \).

Related work: Many routing schemes and lower bounds for the resources required for routing were presented in the past (cf. [16]). The first studies attempting to characterize and bound the resource tradeoffs involved in routing schemes for general networks were presented in [17] and were followed by a number of improved constructions (cf. [5],[10],[11],[12],[19]). These studies focused on routing schemes with compact routing tables and low stretch factors.