Computing the Diameter of 17-Pancake Graph Using a PC Cluster

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Abstract. An n-pancake graph is a graph whose vertices are the permutations of n symbols and each pair of vertices are connected with an edge if and only if the corresponding permutations can be transitive by a prefix reversal. Since the n-pancake graph has $n!$ vertices, it is known to be a hard problem to compute its diameter by using an algorithm with the polynomial order of the number of vertices. Fundamental approaches of the diameter computation have been proposed. However, the computation of the diameter of 15-pancake graph has been the limit in practice. In order to compute the diameters of the larger pancake graphs, it is indispensable to establish a sustainable parallel system with enough scalability. Therefore, in this study, we have proposed an improved algorithm to compute the diameter and have developed a sustainable parallel system with the Condor/MW framework, and computed the diameters of 16- and 17-pancake graphs by using PC clusters.

1 Introduction

In this paper, let us consider a problem in which a stack of pancakes whose sizes are completely different is rearranged so that the pancakes form a pile where the sizes of pancakes increase from the top to the bottom. As operations of rearrangement, reversing several pancakes from the top of the stack is possible. The problem to obtain the largest number of operations to rearrange the worst-case stack of n pancakes as a function of n is called the pancake sorting problem[1]. This problem is also called the prefix reversal problem.

A pancake graph is a graph whose vertices are the permutations of n symbols from 1 to n and its edges are given between permutations transitive by prefix reversals. Since the graph topology is dependent on n, it is called an n-pancake graph. An n-pancake graph is a regular graph that has $n!$ vertices and its degree is $n-1$. The pancake sorting problem and the problem to obtain the diameter of the pancake graph is equivalent. Since the pancake graphs have many merits such as the symmetric and recursive structures, and the small degrees and diameters against the sizes, much attention is paid to them as a model of interconnection networks for parallel computers[2,3,4]. When we regard the pancake graphs as the model of the interconnection networks, the diameter of the graph is a measure that represents the delay of communication[5,6].

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Table 1. The diameters of n-pancake graphs

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Diameters	0	1	3	4	5	7	8	9	10	11	13	14	15	16	17

To obtain the diameter of an n-pancake graph, it is sufficient to obtain the shortest distances from one vertex to all the vertices. However, the algorithms that depend on the numbers of vertices and/or edges cannot solve the problem practically because the computational time and the memory space increase exponentially. Hence, Kounoike et *al.*[7] proposed a method that restricts the number of vertices for which the shortest distances must be obtained by taking advantage of the recursive structure of the pancake graphs. This method is based on the method by Heydari et *al.*[8] to obtain the diameter of the 13-pancake graph and is extended not to execute the unnecessary search. Kounoike has applied the method to give the diameters of 14- and 15-pancake graphs that were unknown so far. Table 1 shows the known diameters of the pancake graphs. Some attentions are paid to the sequence of diameters mathematically, and the sequence up to $n = 13$ is listed in the 'On-Line Encyclopedia of Integer Sequences'[9] as 'Sorting by prefix reversal.' However, no sequence for $n \geq 14$ is listed there. Hence, obtaining the diameters of the larger pancake graphs also contributes the study of the sequences.

In this study, we have improved the method by Kounoike et *al.* when they obtained the diameter of 15-pancake graph so that it computes the diameters of the larger pancake graphs and implemented it as a parallel computing system. In addition, we made use of the implemented system to obtain the diameters of 16- and 17-pancake graphs that have been unknown.

2 Definitions of Terminology and Symbols

In this section, we define the terminology and symbols used in this paper. Refer [7] for the detailed explanations.

Let S_n be the set of all the permutations of n symbols from 1 to n, and let the symbols 1 to n correspond to the smallest size of pancake to the largest one. Then assume that a permutation $\pi \in S_n$ which is obtained by arranging the symbols from the top pancake to the bottom pancake represents a stack of n pancakes. Let $\boldsymbol{e}_n$ be the permutation $(1, 2, \ldots, n)$ that corresponds to the sorted stack. Let $\sigma \in S_n$ be a permutation that is obtained by reversing the preceding k $(2 \leq k \leq n)$ symbols in $\pi \in S_n$. Then the transformation from the permutation π to the permutation σ is called the prefix reversal of k symbols for the permutation π, and it is denoted $\pi^k = \sigma$. Since we use only the prefix reversals of permutations in this paper, we mention reversals to mean the prefix ones. The successive reversals $(\pi^{x_1})^{x_2}$ of a permutation π are also denoted $\pi^{(x_1,x_2)}$. Moreover, if $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$ then let $\pi^{\boldsymbol{x}}$ represent a successive reversals with x_1, $x_2, \ldots, x_m$ symbols. If $\pi^{\boldsymbol{x}} = \boldsymbol{e}_n$ then $\boldsymbol{x}$ is called a sorting sequence of π. For a given permutation $\pi \in S_n$, let the function $f(\pi) = \min\{|\boldsymbol{x}| : \pi^{\boldsymbol{x}} = \boldsymbol{e}_n\}$