Arrow Decision Logic for Relational Information Systems*

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Abstract. In this paper, we propose an arrow decision logic (ADL) for relational information systems (RIS). The logic combines the main features of decision logic (DL) and arrow logic (AL). DL represents and reasons about knowledge extracted from decision tables based on rough set theory, whereas AL is the basic modal logic of arrows. The semantic models of DL are functional information systems (FIS). ADL formulas, on the other hand, are interpreted in RIS. RIS , which not only specifies the properties of objects, but also the relationships between objects. We present a complete axiomatization of ADL and discuss its application to knowledge representation in multicriteria decision analysis.

Keywords: Arrow logic, decision logic, functional information systems, multicriteria decision analysis, relational information systems, rough sets.

1 Introduction

The rough set theory proposed by Pawlak [25] provides an effective tool for extracting knowledge from data tables. To represent and reason about extracted knowledge, a decision logic (DL) is proposed in Pawlak [26]. The semantics of the logic is defined in a Tarskian style through the notions of models and satisfaction. While DL can be considered as an instance of classical logic in the context of data tables, different generalizations of DL corresponding to some non-classical logics are also desirable from the viewpoint of knowledge representation. For

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example, to deal with uncertain or incomplete information, some generalized decision logics have been proposed [7,22,23,34,35].

In rough set theory, objects are partitioned into equivalence classes based on their attribute values, which are essentially functional information associated with the objects. Though many databases contain only functional information about objects, data about the relationships between objects has become increasingly important in decision analysis. A remarkable example is social network analysis, in which the principal types of data are attribute data and relational data.

To represent attribute data, a data table in rough set theory consists of a set of objects and a set of attributes, where each attribute is considered as a function from the set of objects to the domain of values for the attribute. Hence, such data tables are also called *functional information systems* (FIS), and rough set theory can be viewed as a theory of *functional granulation*. Recently, granulation based on relational information between objects, called *relational granulation*, has been studied by Liau and Lin [21]. To facilitate further study of relational granulation, it is necessary to represent and reason about data in *relational information systems* (RIS).

In FIS, the basic entities are objects, while DL formulas describe the properties of such objects, thus, the truth values of DL formulas are evaluated with respect to these objects. To reason about RIS, we need a language that can be interpreted in the domain of pairs of objects, since relations can be seen as properties of such pairs. Arrow logic (AL) [24,33] fulfills this need perfectly. Hence, in this paper, we propose arrow decision logic (ADL), which combines the main features of DL and AL, to represent the decision rules induced from RIS. The atomic formulas of ADL have the same descriptor form as those in DL; while the formulas of ADL are interpreted with respect to each pair of objects, just as in the pair frame of AL [24,33]. The semantic models of ADL are RIS; thus, ADL can represent knowledge induced from systems containing relational information.

The remainder of this paper is organized as follows. In Section 2, we review FIS in rough set theory and give a precise definition of RIS. We study the relationship between these two kinds of information system and present some practical examples. In Section 3, we review DL and AL to lay the foundation for ADL. In Section 4, we present the syntax and semantics of ADL. A complete axiomatization of ADL based on the combination of DL and AL axiomatic systems is presented. In Section 5, we define some quantitative measures for the rules of ADL and discuss the application of ADL to data analysis. Finally, we present our conclusions in Section 6.

2 Information Systems

Information systems are fundamental to rough set theory, in which the approximation space can be derived from attribute-value information systems [26]. In this section, we review the functional information systems used in the original rough set theory and propose a generalization of it, namely, relational