On Generalized Rough Fuzzy Approximation Operators

Wei-Zhi Wu[1], Yee Leung[2], and Wen-Xiu Zhang[3]

[1] Information College, Zhejiang Ocean University,
Zhoushan, Zhejiang, 316004, P.R. China
wuwz@zjou.net.cn

[2] Department of Geography and Resource Management
Center for Environmental Policy and Resource Management
and Institute of Space and Earth Information Science
The Chinese University of Hong Kong, Hong Kong
yeeleung@cuhk.edu.hk

[3] Institute for Information and System Sciences, Faculty of Science
Xi'an Jiaotong University, Xi'an, Shaan'xi, 710049, P.R. China
wxzhang@mail.xjtu.edu.cn

Abstract. This paper presents a general framework for the study of rough fuzzy sets in which fuzzy sets are approximated in a crisp approximation space. By the constructive approach, a pair of lower and upper generalized rough fuzzy approximation operators is first defined. The rough fuzzy approximation operators are represented by a class of generalized crisp approximation operators. Properties of rough fuzzy approximation operators are then discussed. The relationships between crisp relations and rough fuzzy approximation operators are further established. By the axiomatic approach, various classes of rough fuzzy approximation operators are characterized by different sets of axioms. The axiom sets of rough fuzzy approximation operators guarantee the existence of certain types of crisp relations producing the same operators. The relationship between a fuzzy topological space and rough fuzzy approximation operators is further established. The connections between rough fuzzy sets and Dempster-Shafer theory of evidence are also examined. Finally multi-step rough fuzzy approximations within the framework of neighborhood systems are analyzed.

Keywords: approximation operators, belief functions, binary relations, fuzzy sets, fuzzy topological spaces, neighborhood systems, rough fuzzy sets, rough sets.

1 Introduction

The theory of rough sets was originally proposed by Pawlak [26,27] as a formal tool for modelling and processing incomplete information. The basic structure of the rough set theory is an approximation space consisting of a universe of discourse and an equivalence relation imposed on it. The equivalence

J.F. Peters and A. Skowron (Eds.): Transactions on Rough Sets V, LNCS 4100, pp. 263–284, 2006.

relation is a key notion in Pawlak's rough set model. The equivalence classes in Pawlak's rough set model provide the basis of "information granules" for database analysis discussed in Zadeh's [67,68]. Rough set theory can be viewed as a crisp-set-based granular computing method that advances reaearch in this area [12,17,29,30,37,38,62].

However, the requirement of an equivalence relation in Pawlak's rough set model seems to be a very restrictive condition that may limit the applications of the rough set model. Thus one of the main directions of research in rough set theory is naturally the generalization of the Pawlak rough set approximations. There are at least two approaches for the development of rough set theory, namely the constructive and axiomatic approaches. In the constructive approach, binary relations on the universe of discourse, partitions of the universe of discourse, neighborhood systems, and Boolean algebras are all the primitive notions. The lower and upper approximation operators are constructed by means of these notions [15,23,25,26,27,28,31,39,40,46,53,56,58,59,60,61,63]. Constructive generalizations of rough set to fuzzy environment have also been discussed in a number of studies [1,2,9,10,14,19,20,21,32,50,51,52,54,57]. For example, by using an equivalence relation on U, Dubois and Prade introduced the lower and upper approximations of fuzzy sets in a Pawlak approximation space to obtain an extended notion called rough fuzzy set [9,10]. Alternatively, a fuzzy similarity relation can be used to replace an equivalence relation. The result is a deviation of rough set theory called fuzzy rough set [10,21,32,58]. Based on arbitrary fuzzy relations, fuzzy partitions on U, and Boolean subalgebras of $\mathcal{P}(U)$, extended notions called rough fuzzy sets and fuzzy rough sets have been obtained [19,20,23,50,51,52,54]. Alternatively, a rough fuzzy set is the approximation of a fuzzy set in a crisp approximation space. The rough fuzzy set model may be used to handle knowledge acquisition in information systems with fuzzy decisions [70]. And a fuzzy rough set is the approximation of a crisp set or a fuzzy set in a fuzzy approximation space. The fuzzy rough set model may be used to unravel knowledge hidden in fuzzy decision systems [55]. Employing constructive methods, extensive research has also been carried out to compare the theory of rough sets with other theories of uncertainty such as fuzzy sets and conditional events [3,19,24,46]. Thus the constructive approach is suitable for practical applications of rough sets.

On the other hand, the axiomatic approach, which is appropriate for studying the structures of rough set algebras, takes the lower and upper approximation operators as primitive notions. From this point of view, rough set theory may be interpreted as an extension theory with two additional unary operators. The lower and upper approximation operators are related respectively to the necessity (box) and possibility (diamond) operators in modal logic, and the interior and closure operators in topological space [4,5,6,13,22,44,45,47,48,60,63]. By this approach, a set of axioms is used to characterize approximation operators that are the same as the ones produced by using the constructive approach. Zakowski [69] studied a set of axioms on approximation operators. Comer [6] investigated axioms on approximation operators in relation to cylindric algebras. The