Rough Belief Change*

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Abstract. The article aims at re-visiting the notion of *rough truth* proposed by Pawlak in 1987 [15] and investigating some of its 'logical' consequences. We focus on the formal deductive apparatus $\mathcal{L}_{\mathcal{R}}$, that is sound and complete with respect to a semantics based on rough truth. $\mathcal{L}_{\mathcal{R}}$ turns out to be equivalent to the paraconsistent logic J due to Jaśkowski. A significant feature of rough truth is that, a proposition and its negation may well be roughly true together. Thus, in [5], *rough consistency* was introduced. Completeness of $\mathcal{L}_{\mathcal{R}}$ is proved with the help of this notion of consistency. The properties of $\mathcal{L}_{\mathcal{R}}$ motivate us to use it for a proposal of *rough belief change*. During change, the operative constraints on a system of beliefs are that of *rough* consistency preservation and deductive closure with respect to $\mathcal{L}_{\mathcal{R}}$. Following the AGM [1] line, eight basic postulates for defining rough revision and contraction functions are presented. Interrelationships of these functions are also proved. The proposal is, therefore, an example of paraconsistent belief change.

1 Introduction

The notion of *rough truth* was introduced in [15] as a part of the first formal proposal on reasoning with rough sets. The work in [15], in fact, paved the way for much subsequent study on logics of rough sets, a good survey of which can be found in [11]. But rough truth seems to have escaped due attention, though it was developed to some extent in [5,3]. The present article investigates related issues and some further 'logical' implications of this notion.

Rough truth was proposed to reflect 'inductive' truth, i.e. truth relative to our present state of knowledge, and one that, with gain of knowledge, leads to total, 'deductive' truth. This sense of 'gradualness' finds an expression in, possibly, the only *qualitative* version of 'approximate' or 'soft' truth, as opposed to other quantitative definitions found in, e.g., probabilistic, multi-valued or fuzzy logics. Let us look at the definition formally.

It has generally been accepted that the propositional aspects of Pawlak's rough set theory are adequately expressed by the modal system $S5$. An $S5$ (Kripke) model (X, R, π) (cf. e.g. [9]) is essentially an *approximation space* [14]

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(X, R), where $X \neq \emptyset$, with the function π interpreting every well-formed formula (wff) of $S5$ as a rough set in (X, R). If L, M denote the necessity and possibility connectives respectively, a *modal wff* $L\alpha$ ($M\alpha$), representing 'definitely' ('possibly') α, is interpreted by π as the lower (upper) approximation $\underline{\pi(\alpha)}$ ($\overline{\pi(\alpha)}$) of the set $\pi(\alpha)$.

Using this formalism, a wff α may be termed *roughly true* in (X, R, π), if $\overline{\pi(\alpha)} = X$. In [5,3], we extended rough truth to *rough validity*, and also introduced the notions of *rough consequence*, *rough (in)consistency*. These were further considered in the context of predicate logic in [4]. The rationale behind the introduction of the concepts was as follows.

Given the aforementioned syntax, one may wish to derive in it, roughly true propositions/beliefs from roughly true premisses (in the same information system). In particular, one may look for interderivability of propositions that are both roughly true and logically equivalent in possibility. This led to the relation of rough consequence. It was also felt that the notion of (in)consistency needs to be relaxed. In the face of an incomplete description of a concept p, one may not always think that p and 'not' p represent conflicting situations. There could be two options to define consistency here – according as 'possibly' p is satisfied, and 'necessarily' p is satisfied. It is thus that we have the two notions of rough consistency and rough inconsistency.

In this paper, we focus on these features of rough reasoning again, and on the syntactic counterpart $\mathcal{L}_{\mathcal{R}}$ of a semantics based on rough truth. $\mathcal{L}_{\mathcal{R}}$ is built over $S5$, and is a modified version of the rough consequence logic of [5]. One observes that the logic is *paraconsistent*, i.e. if a set Γ of wffs in it contains two members, one of which is the negation of the other, then Γ does *not* yield all wffs as its consequence. In other words, it violates the principle of *ex contradictione sequitur quodlibet* (*ECQ*), viz. $\Gamma \cup \{\alpha, \neg\alpha\} \models \beta$, for all Γ, α, β. In fact, $\mathcal{L}_{\mathcal{R}}$ is seen to be equivalent to the paraconsistent logic J due to Jaśkowski (cf. e.g. [6], implicitly present in [10]). We present the system $\mathcal{L}_{\mathcal{R}}$ and its properties in Section 2. Proofs of the main results stated in the section are given in the Appendix.

Research on belief change has seen a lot of development through the years. A pioneering work, undoubtedly, has been by Alchourrón, Gärdenfors and Makinson in [1]. The formalisation propounded by the authors consists of three main kinds of belief change: *expansion, revision, and contraction*. In the first kind, a belief is inserted into a system S (say), irrespective of whether S becomes 'inconsistent' as a result. Revision and contraction represent, respectively, insertion and deletion of beliefs maintaining ('integrity' constraints of) consistency of the system, deductive closure and minimal information loss. The AGM 'rationality' postulates for defining revision and contraction functions were formulated in [1]. A to-and-fro passage between contraction and revision is established through the *Levi* and *Harper identities*.

The AGM postulates have since been questioned, modified, and alternatives like algorithms for computing the change functions have been proposed