Rough Sets and Vague Concept Approximation: From Sample Approximation to Adaptive Learning

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Abstract. We present a rough set approach to vague concept approximation. Approximation spaces used for concept approximation have been initially defined on samples of objects (decision tables) representing partial information about concepts. Such approximation spaces defined on samples are next inductively extended on the whole object universe. This makes it possible to define the concept approximation on extensions of samples. We discuss the role of inductive extensions of approximation spaces in searching for concept approximation. However, searching for relevant inductive extensions of approximation spaces defined on samples is infeasible for compound concepts. We outline an approach making this searching feasible by using a concept ontology specified by domain knowledge and its approximation. We also extend this approach to a framework for adaptive approximation of vague concepts by agents interacting with environments. This paper realizes a step toward approximate reasoning in multiagent systems (MAS), intelligent systems, and complex dynamic systems (CAS).

Keywords: Vagueness, rough sets, approximation space, higher order vagueness, adaptive learning, incremental learning, reinforcement learning, constraints, intelligent systems.

1 Introduction

In this paper, we discuss the rough set approach to vague concept approximation. There has been a long debate in philosophy about vague concepts [18].

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Nowadays, computer scientists are also interested in vague (imprecise) concepts, e.g, many intelligent systems should satisfy some constraints specified by vague concepts. Hence, the problem of vague concept approximation as well as preserving vague dependencies especially in dynamically changing environments is important for such systems. Lotfi Zadeh [66] introduced a very successful approach to vagueness. In this approach, sets are defined by partial membership in contrast to crisp membership used in the classical definition of a set. Rough set theory [32] expresses vagueness not by means of membership but by employing the boundary region of a set. If the boundary region of a set is empty it means that a particular set is crisp, otherwise the set is rough (inexact). The non-empty boundary region of the set means that our knowledge about the set is not sufficient to define the set precisely. In this paper, some consequences on understanding of vague concepts caused by inductive extensions of approximation spaces and adaptive concept learning are presented. A discussion on vagueness in the context of fuzzy sets and rough sets can be found in [40].

Initially, the approximation spaces were introduced for decision tables (samples of objects). The assumption was made that the partial information about objects is given by values of attributes and on the basis of such information about objects the approximations of subsets of objects form the universe restricted to sample have been defined [32]. Starting, at least, from the early 90s, many researchers have been using the rough set approach for constructing classification algorithms (classifiers) defined over extensions of samples. This is based on the assumption that available information about concepts is partial. In recent years, there have been attempts based on approximation spaces and operations on approximation spaces for developing an approach to approximation of concepts over the extensions of samples (see, e.g., [48,50,51,56]). In this paper, we follow this approach and we show that the basic operations related to approximation of concepts on extension of samples are inductive extensions of approximation spaces. For illustration of the approach we use approximation spaces defined in [47]. Among the basic components of approximation spaces are neighborhoods of objects defined by the available information about objects and rough inclusion functions between sets of objects. Observe that searching for relevant (for approximation of concepts) extensions of approximation spaces requires tuning many more parameters than in the case of approximation of concepts on samples. The important conclusion from our considerations is that the inductive extensions used in constructing of algorithms (classifiers) are defined by arguments "for" and "against" of concepts. Each argument is defined by a tuple consisting of a degree of inclusion of objects into a pattern and a degree of inclusion of the pattern into the concepts. Patterns in the case of rule-based classifiers can be interpreted as the left hand sides of decision rules. The arguments are discovered from available data and can be treated as the basic information granules used in the concept approximation process. For any new object, it is possible to check the satisfiability of arguments and select arguments satisfied to a satisfactory degree. Such selected arguments are fused by conflict resolution strategies for obtaining the classification decision. Searching for rel-