Timed Alternating-Time Temporal Logic*

Thomas A. Henzinger[1] and Vinayak S. Prabhu[2]

[1] Department of Computer and Communication Sciences, EPFL
tah@epfl.ch
[2] Department of Electrical Engineering and Computer Sciences, UC Berkeley
vinayak@eecs.berkeley.edu

Abstract. We add freeze quantifiers to the game logic ATL in order to specify real-time objectives for games played on timed structures. We define the semantics of the resulting logic TATL by restricting the players to physically meaningful strategies, which do not prevent time from diverging. We show that TATL can be model checked over timed automaton games. We also specify timed optimization problems for physically meaningful strategies, and we show that for timed automaton games, the optimal answers can be approximated to within any degree of precision.

1 Introduction

Timed games are a formal model for the synthesis of real-time systems [22,20]. While much research effort has been directed at algorithms for solving timed games [14,9,7,16,15,8,11], we find it useful to revisit the topic for two reasons. First, we wish to study a perfectly symmetric setup of the model, where all players (whether they represent a plant, a controller, a scheduler, etc.) are given equally powerful options for updating the state of the game, advancing time, or blocking time. Second, we wish to restrict all players to physically meaningful strategies, which do not allow a player to prevent time from diverging in order to achieve an objective. This restriction is often ensured by syntactic conditions on the cycles of timed automaton games [7,16,8,21] or by semantic conditions that discretize time; we find such conditions unsatisfactory and unnecessary: unsatisfactory, because they rule out perfectly meaningful strategies that suggest an arbitrary but finite number of transitions in a finite interval of time; unnecessary, because timed automaton games can be solved without such conditions.

We do not present a new model for timed games, but review the model of [13], which is symmetric for all players and handles the divergence of dense time without constraining the players. We consider the two-player case. Previous work on the existence of controllers [14,9,20,11] has in general required that time divergence be *ensured* by the controller — an unfair view in settings where the player for the environment can also block time. In our model, both players may block time, however, for a player to win for an objective, she must not be *responsible* for preventing time from diverging. To achieve this, we distinguish

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between *objectives* and *winning conditions*. An objective for a player is a set Φ of desired outcomes of the game. The winning condition WC maps the objective to another set of outcomes so that the player wins for $\mathsf{WC}(\Phi)$ using any strategy if and only if she wins for the original objective Φ using a physically meaningful strategy.

Let us be more precise. A timed game proceeds in an infinite sequence of turns. At each turn, both players propose a move: each move specifies an amount of time that the player is willing to let pass without action, possibly followed by an action that causes a discontinuous jump to a different state. The move with the *shorter* proposed time delay determines the next state of the game (if both players propose the same delays, then one of the corresponding actions is chosen nondeterministically). An outcome of the game is an infinite trajectory of continuous state segments (during which time passes) and discontinuous jumps. Let $\mathsf{Timediv}$ denote the outcomes for which time diverges (the other trajectories are often called “zeno” behaviors). Let $\mathsf{Blameless}_i$ denote the outcomes in which player $i \in \{1, 2\}$ proposes the shorter delay only finitely often. Clearly, player i is not responsible if time converges for an outcome in $\mathsf{Blameless}_i$. We therefore use the winning condition [13]

$$\mathsf{WC}_i(\Phi) \;=\; (\mathsf{Timediv} \cap \Phi) \;\cup\; (\mathsf{Blameless}_i \setminus \mathsf{Timediv}).$$

Informally, this condition states that if an outcome is time divergent, then it is a valid outcome, and hence must satisfy the objective Φ; and if it is not time divergent, then player i must not be responsible for the zeno behaviour. The winning conditions for both players are perfectly symmetric: since $\mathsf{WC}_1(\Phi) \cap \mathsf{WC}_2(\neg\Phi) = \emptyset$, at most one player can win.

In [6], several alternating-time temporal logics were introduced to specify properties of game structures, including the CTL-like logic ATL, and the CTL*-like logic ATL*. For example, the ATL formula $\langle\langle i \rangle\rangle \Diamond p$ is true at a state s iff player i can force the game from s into a state that satisfies the proposition p. We interpret these logics over *timed* game structures, and enrich them by adding *freeze* quantifiers [5] for specifying timing constraints. The resulting logics are called TATL and TATL*. The new logic TATL subsumes both the untimed game logic ATL, and the timed non-game logic TCTL [3]. For example, the TATL formula $\langle\langle i \rangle\rangle \Diamond_{\leq d}\, p$ is true at a state s iff player i can force the game from s into a p state in at most d time units. A version of TATL has recently been studied on durational concurrent structures in [19] .

The model checking of these logics requires the solution of timed games. Timed game structures are infinite-state. In order to consider algorithmic solutions, we restrict our attention to timed game structures that are generated by a finite syntax borrowed from timed automata [4]. By restricting the strategies of TATL games to physically meaningful strategies using WC, we obtain TATL* games. However, solving TATL* games is undecidable, because TATL* subsumes the linear-time logic TPTL [5], whose dense-time satisfiability problem is undecidable. We nontheless establish the decidability of TATL model checking, by carefully analyzing the fragment of TATL* we obtain through the WC translation.