Undecidable Problems About Timed Automata

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Abstract. We solve some decision problems for timed automata which were raised by S. Tripakis in [Tri04] and by E. Asarin in [Asa04]. In particular, we show that one cannot decide whether a given timed automaton is determinizable or whether the complement of a timed regular language is timed regular. We show that the problem of the minimization of the number of clocks of a timed automaton is undecidable. It is also undecidable whether the shuffle of two timed regular languages is timed regular. We show that in the case of timed Büchi automata accepting infinite timed words some of these problems are Π_1^1-hard, hence highly undecidable (located beyond the arithmetical hierarchy).[1]

Keywords: Timed automata; timed Büchi automata; timed regular (ω)-languages; decision problems; universality problem; determinizability; complementability; shuffle operation; minimization of the number of clocks.

1 Introduction

R. Alur and D. Dill introduced in [AD94] the notion of timed automata reading timed words. Since then the theory of timed automata has been much studied and used for specification and verification of timed systems.

In a recent paper, E. Asarin raised a series of questions about the theoretical foundations of timed automata and timed languages which were still open and wrote: "I believe that getting answers to them would substantially improve our understanding of the area" of timed systems, [Asa04].

Some of these questions concern decision problems "à la [Tri04]". For instance : "Is it possible, given a timed automaton $\mathcal{A}$, to decide whether it is equivalent to a deterministic one ?".

S. tripakis showed in [Tri04] that there is no algorithm which, given a timed automaton $\mathcal{A}$, decides whether it is equivalent to a deterministic one, **and** if this is the case gives an equivalent deterministic automaton $\mathcal{B}$. But the above

[1] Part of the results stated in this paper were presented very recently in the Bulletin of the EATCS [Fin05, Fin06].

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question of the decidability of the determinizability alone (where we do not require the construction of the witness $\mathcal{B}$) was still open.

We give in this paper the answer to this question and to several other ones of [Tri04, Asa04]. In particular, we show that one cannot decide whether a given timed automaton is determinizable or whether the complement of a timed regular language is timed regular. We study also the corresponding problems but with "bounded resources" stated in [Tri04].

For that purpose we use a method which is very similar to that one used in [Fin03b] to prove undecidability results about infinitary rational relations, reducing the universality problem, which is undecidable, to some other decision problems.

We study also the problem of the minimization of the number of clocks of a timed automaton, showing that one cannot decide, for a given timed automaton $\mathcal{A}$ with n clocks, $n \geq 2$, whether there is an equivalent timed automaton $\mathcal{B}$ with at most $n-1$ clocks.

The question of the closure of the class of timed regular languages under shuffle was also raised by E. Asarin in [Asa04]. C. Dima proved in [Dim05] that timed regular expressions with shuffle characterize timed languages accepted by stopwatch automata. This implies that the class of timed regular languages is not closed under shuffle. We proved this result independently in [Fin06]. We recall the proof here, giving a simple example of two timed regular languages whose shuffle is not timed regular. Next we use this example to prove that one can not decide whether the shuffle of two given timed regular languages is timed regular.

We extend also the previous undecidability results to the case of timed Büchi automata accepting infinite timed words. In this case many problems are Π_1^1-hard, hence highly undecidable (located beyond the arithmetical hierarchy), because the universality problem for timed Büchi automata, which is itself Π_1^1-hard, [AD94], can be reduced to these other decision problems.

We mention that part of the results stated in this paper were presented very recently in the Bulletin of the EATCS [Fin05, Fin06].

The paper is organized as follows. We recall usual notations in Section 2. The undecidability of determinizability or regular complementability for timed regular languages is proved in Section 3. The problem of minimization of the number of clocks is studied in Section 4. Results about the shuffle operation are stated in Section 5. Finally we extend in Section 6 some undecidability results to the case of timed Büchi automata.

2 Notations

We assume the reader to be familiar with the basic theory of timed languages and timed automata (TA) [AD94].

The set of positive reals will be denoted $\mathcal{R}$. A (finite length) timed word over a finite alphabet Σ is of the form $t_1.a_1.t_2.a_2 \ldots t_n.a_n$, where, for all integers $i \in [1, n]$, $t_i \in \mathcal{R}$ and $a_i \in \Sigma$. It may be seen as a *time-event sequence*, where