A Dose of Timed Logic, in Guarded Measure

Kamal Lodaya[1] and Paritosh K. Pandya[2]

[1] The Institute of Mathematical Sciences
CIT Campus, Chennai 600113, India
[2] Tata Institute of Fundamental Research
Colaba, Mumbai 400005, India

Abstract. We consider interval measurement logic *IML*, a sublogic of Zhou and Hansen's interval logic, with measurement functions which provide real-valued measurement of some aspect of system behaviour in a given time interval. We interpret *IML* over a variety of time domains (continuous, sampled, integer) and show that it can provide a unified treatment of many diverse temporal logics including duration calculus (DC), interval duration logic (IDL) and metric temporal logic (MTL). We introduce a fragment *GIML* with restricted measurement modalities which subsumes most of the decidable timed logics considered in the literature.

Next, we introduce a guarded first-order logic with measurements *MGF*. As a generalisation of Kamp's theorem, we show that over arbitrary time domains, the measurement logic *GIML* is expressively complete for it. We also show that *MGF* has the 3-variable property.

In addition, we have a preliminary result showing the decidability of a subset of *GIML* when interpreted over timed words.

The importance of reasoning about timed systems has led to considerable research on models and logics for timed behaviours. We consider a slightly more general situation where, in addition to time, we can use other measurement functions as well. For instance, instead of saying "during the last 24 hours, the rainfall was 100 mm," we can say that "the time elapsed for the last 100 mm of rainfall was over 4 months." We can also have measurements of quantities like "mean value" of a proposition within a time interval. Guelev has shown how probabilities might be incorporated into such a framework [Gue00].

Unlike data languages [BPT03], there is no finite state mechanism associated with the measurement functions. Thus we are in the setting of the interval logic with measurements defined by Zhou and Hansen [ZH04].

There exists quite a menagerie of timed and duration logics. In Section 1 below, we review the literature and define our logic $\chi IML[\Sigma]$ over a signature Σ of measurement functions, and parameterised by a set of primitive comparisons χ dependent on Σ. We show that it can provide a unified treatment of many diverse temporal logics including duration calculus (DC), interval duration logic (IDL) and metric temporal logic (MTL).

In Section 2, we consider an enrichment of Kamp's $FO[<]$ with measurements. The undecidability of this logic motivates us to formulate and investigate a

E. Asarin and P. Bouyer (Eds.): FORMATS 2006, LNCS 4202, pp. 260–273, 2006.

fragment $\chi MGF[<, \Sigma]$ with χ-guarded measurement quantifiers. The next two sections show that $\chi GIML$ is expressively complete for χMGF. Kamp's syntactic techniques were used by Venema [Ven91], and we extend these as well as the pebble games of Immerman and Kozen [IK87] in our proofs. As in Kamp's result, we show along the way that χMGF has the three-variable property.

Thus the expressiveness of our logic is reasonably delineated. We would have liked to have established a connection to aperiodic languages [Bac03] but this must remain future work.

We now turn to decidability. We find that *IML* and *GIML* are in general undecidable, but for a set of weak comparisons (which disallow equality tests between measurements and constants), we use a result by Hirshfeld and Rabinovich [HR99] and our expressive completeness to show that *Weak-GIML*$[\ell]$ is decidable for continous time. We also prove by translation into one-clock alternating timed automata [LW05, OW05], decidability over timed words of a sublogic *Punct-FgIML*$[\ell]$ of *GIML*$[\ell]$, which only has nesting-free forward guarding.

1 A Classification of Timed Behaviours and Logics

Timed logics describe the evolution of system behaviour in time. For us, time is a linear order $(T, <)$, and we will further assume that T is a subset of the non-negative reals (which we designate $\Re$) with $<$ the usual ordering. $Intv(T)$, the set of (closed) intervals of T, is $\{[b, e] \in T \times T \mid b \leq e\}$. A time frame $TF = (T, <, d)$ is a subset of the real order $(\Re, <)$ with d giving the absolute value of the distance on the real line between two real numbers, i.e. $d[b, e] = |b - e|$.

Zhou and Hansen have proposed an interesting interval logic [ZH04] where the variables (measurement functions) denote real-valued measurements of system behaviour in a given time interval. Formally we have a signature $\Sigma = \{m_1, \ldots, m_n\}$ of measurement function symbols (of arity 2), and we assume that it contains the distinguished function ℓ which measures the length of the interval. We will often abbreviate the signature $\{\ell\}$ to ℓ.

Zhou and Hansen's logic allows first order real arithmetic over such measurements. In this section, we introduce a restricted version of this logic where a measurement may only be compared with an integer constant. We call this logic **interval measurement logic**, *IML*.

Let $Pvar$ be a finite set of propositional variables. A behaviour of a system over TF is a pair of maps $\theta : (Pvar \to T \to \{0, 1\}) \times (\Sigma \to Intv \to \Re)$, where Σ might depend on $Pvar$. For convenience we write $\theta(p)$ as a boolean function of time and $\theta(m)[b, e]$, for $m \in \Sigma$, as giving the value of the measurement function m on the interval $[b, e]$. Moreover, we require that the measurement ℓ is always interpreted as length of the interval, i.e. $\theta(\ell)[b, e] = d[b, e] = |b - e|$. An interval model is a pair $\theta, [b, e]$.

It is useful to consider several classes of time frames $TF = (T, <, d)$ where $T \subseteq \Re$. In the literature, we find a variety of timed logics which use these different classes as time frames. Some such logics are listed in the next section.