

# Bridging the Gap Between Timed Automata and Bounded Time Petri Nets

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**Abstract.** Several recent papers investigate the relative expressiveness of Timed Automata and Time Petri Nets, two widespread models for realtime systems. It has been shown notably that Timed Automata and Bounded Time Petri Nets are equally expressive in terms of timed language acceptance, but that Timed Automata are strictly more expressive in terms of weak timed bisimilarity. This paper compares Timed Automata with Bounded Time Petri Nets extended with static Priorities, and shows that two large subsets of these models are equally expressive in terms of weak timed bisimilarity.

**Keywords:** Time Petri nets, priorities, Timed Automata, weak timed bisimilarity, real-time systems modeling and verification.

## 1 Introduction

Among the many models proposed for the specification and verification of real time systems, two are prominent: Time Petri nets and Timed Automata.

Time Petri nets (*TPN*) [15] extend Petri nets with temporal intervals associated with transitions, specifying firing delay ranges for the transitions. Assuming transition  $t$  became last enabled at time  $\theta$ , and the end-points of its time interval are  $\alpha$  and  $\beta$ , then  $t$  cannot fire earlier than time  $\theta + \alpha$  and must fire no later than  $\theta + \beta$ , unless disabled by firing some other transition. Firing a transition takes no time. Many other Petri net based models with time extensions have been proposed, but none reaches the acceptance of Time Petri nets. Availability of effective analysis methods, prompted by [5], certainly contributed to their widespread use, together with their ability to cope with a wide variety of modeling problems for realtime systems.

Timed Automata (*TA*) [2] extend finite automata with clocks, guards, resets and a product operation. Transitions are guarded by boolean conditions on clock values. When taken, they may emit a label and perform resets of some clocks. All clocks progress synchronously as time elapses. Some versions of timed automata support progress annotations as location invariants limiting elapsing of time when at that location, urgency requirements, or transition deadlines. Timed automata are convenient for modeling a large class of realtime problems. They prompted a considerable amount of research work and benefit from a rich theory.

These two models, as well as their analysis techniques, were developed independently for years, though they bear strong relationships. State space abstractions for *TPN*'s preserving various classes of properties can be computed in terms of so-called state classes [5] [4] [6]. State classes represent sets of states by a marking and a polyhedron capturing temporal information. State space abstractions for (Networks of) Timed Automata are based upon geometric regions characterizing sets of states by a location of the underlying automaton and a convex set capturing temporal information. In both cases, the convex sets can be represented by difference systems, or DBM's.

In spite of many technical resemblances and their overlapping application domains, few material was available until recently comparing expressiveness of these two models. A number of recent works finally addressed the issue. [11] translated a subclass of *TA*'s into *TPN*'s, preserving timed language acceptance. Later, [9] proposed a structural encoding of *TPN*'s into *TA*'s, improving an earlier semantics based encoding in [14]. [3] proves that *TPN*'s and *TA*'s are equivalent w.r.t. timed language acceptance, but that *TA*'s are strictly more expressive in terms of timed bisimilarity, they also discuss the subclass of *TA*'s weakly timed bisimilar with some *TPN*.

In this article, we first extend Time Petri nets with static priorities. In *TPN*'s with Priorities (*PrTPN*'s for short), a transition is not allowed to fire if some transition with higher priority is fireable at the same instant. Such priorities have many applications in realtime systems, in scheduling, arbitration, synchronization, and others problems. We then develop an encoding of Timed Automata (without progress requirements) into *PrTPN*'s, preserving weak timed bisimilarity. Next, we extend *TA*'s with invariants and show that *TA*'s with invariants built from  $\{\leq, \wedge\}$  can be encoded into *PrTPN*'s. Finally, extending the encoding of [9] of *TPN*'s into *TA*'s, we show that *TA*'s with invariants built from  $\{\leq, \wedge\}$  are equally expressive than *PrTPN*'s with unbounded or right-closed intervals. Some corollaries follow that extend available equivalence results between *TA*'s and *TPN*'s (without priorities).

The paper is organized as follows. Section 2 recalls the essentials about timed transition systems (*TTS*), the common semantic domain for *TA*'s and *PrTPN*'s. Section 3 reviews the terminology of Timed automata and their semantics. Section 4 introduces Time Petri nets with Priorities, and compares their expressiveness with that of *TPN*'s. Section 5 explains how to encode Timed Automata (without invariants) into weakly timed bisimilar *PrTPN*'s. Section 6 discusses progress requirements, extends the encoding to *TA*'s with invariants built from  $\{\leq, \wedge\}$ , and derives a number of ordering or equivalence results. Finally, Section 7 discusses some consequences, side issues, and prospective work.

## 2 Timed Transition Systems

The semantics of Timed Automata (*TA*) and Time Petri Nets (*PrTPN*) will be given in terms of Timed Transition Systems (*TTS*), as described in e.g. [13]. We review here their terminology and some key concepts used in the next sections.