The Reasoning and Analysis of Spatial Direction Relation Based on Voronoi Diagram

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Abstract. This paper studies the direction relations between two spatial objects and presents the reasoning model of spatial direction relation based on Voronoi diagram. The algorithm of the reasoning is described detailedy and an example is analyzed. This paper also discusses the six typical circumstances about spatial direction relation based on Voronoi diagram. These include the direction relations of point-point, point-line, point-region, line-line, line-region and region-region. At last, the advantages and disadvantages about the model are summarized.

1 Introduction

Spatial relations mainly include topology, direction and distance relations. Spatial topological relation is a kind of topological invariant by topological transformation such as adjacency and connectivity relations. Direction relation is also a kind of order relations. It describes a certain order of spatial objects such as forward and back, up and down, left and right, east, south, west, north and so on. Distance relation means the distance between two spatial objects. There are a lot of the research which relate to distance and topology. But the research of direction relation lags behind oppositely. Up to now, the models of describing spatial direction relation mainly have seven categories: Cone-Based model [1], Projection-Based model [1], Double-Cross model [2], 2D String model [3], MBR (Minimal Bounding Rectangle) model [4, 5], Direction Relation Matrix model [6], Voronoi-Based MBR model [7]. Thereinto, Cone-Based model and Projection-Based model belong to experience models. Double-Cross model is on the basis of Projection-Based model. The three models are used for point objects and can’t be applied to region objects. The basic idea of 2D String model is based on the method of sign projection. The borders of different 2D spatial objects are projected on X axis and Y axis respectively. Spatial relations are expressed and judged by the character strings which come from the projection and have order relations. Therefore, the higher dimension problems are solved by the model which uses the method with one dimension. It is difficult to assure reliability and maturity and it doesn’t express topology relations. It is also difficult to extend 3D space. MBR model and Direction Relation Matrix model describe spatial direction relations intuitively. But for two spatial objects which the diameter ratio is very large, it is not exact to use MBR model and Direction Relation Matrix model to describe spatial direction relations. The basic idea of
Voronoi-Based MBR model is that the four sides of a spatial entity’s MBR are regarded as four growth cells, the four growth cells create four sides’ Voronoi diagram and then the direction relations are formally described by two entities’ Voronoi region and Voronoi boundary. But it also exists the same deficiency as MBR model. Furthermore, it exists very bigger errors for the slope linear objects.

This paper presents a new approach to the problem about spatial direction relation, this approach describes the direction relations between two spatial objects very well. The presented method about spatial direction relation is based on Voronoi diagram.

The rest of the article is structured as follows. In Section 2, the reasoning model of spatial direction relation based on Voronoi diagram is presented and the algorithm about spatial direction relation based on Voronoi diagram is also explained detailedly. Section 3 analyzes the six typical circumstances about spatial direction relation based on Voronoi diagram such as point-point, point-line, point-region, line-line, line-region and region-region. Section 4 is an analysis and evaluation of an example about the algorithm. Finally, conclusions are given in the last section.

2 Algorithm Description of Spatial Direction Relation Based on Voronoi Diagram

The Voronoi region of an entity has a special meaning—the influence region of itself and is defined as the area containing all locations closer to itself than to any other.

[Definition] Voronoi Diagram: suppose having a set of spatial objects, \( P = \{P_1, P_2, P_3, \ldots, P_n\} \subseteq IR^2, \{1 \leq n \leq \infty\} \), \( P_i \) may be a point object \( P_P \), or a line object \( P_L \) or a region object \( P_R \). A region object is not necessarily convex, and may have holes in which another region may exist. The object Voronoi region of \( P_i \) can be defined as \( V(P_i) = \{y|\text{Dist}(y,P_i) \leq \text{Dist}(y,P_j), i \neq j\} \), where \( \text{Dist}(y,P_i) \) means the minimal distance between point \( y \) and \( P_i \). \( V(P_i) \) is the Voronoi polygon of spatial object \( P_i \), \( V(P) = \{V(P_1), V(P_2), \ldots, V(P_n)\} \) is called Voronoi Diagram.

The Voronoi Diagram of point, line and region objects is shown in Figure 1. Indeed, a Voronoi Diagram (Voronoi-Based tessellation) is closer to human perceptions.

Because spatial direction between two objects takes on complexity and variety and there have a lot of directional line segments. A directional line segment means a direction vector which starts from a reference point to a target point. It is impossible to find out each directional line segment one by one. If considering each normal corresponding to each directional line segment, these normal can form Voronoi boundary between two objects (Figure 2), i.e. the curve which \( E_0 \) denotes. So if knowing the Voronoi diagram between two objects, Voronoi boundary is also known, the direction relation is judged by the relation which direction and Voronoi sides are vertical with each other. Consequently, the key problem