A Group Quorum System of Degree $1 + \sqrt{1 + \frac{n}{m}}$

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Abstract. The group mutual exclusion problem is a generalization of the ordinary mutual exclusion problem where each application process can be a member of different groups and members of the same group are allowed simultaneous access to their critical sections. Members of different groups must access their critical sections in a mutually exclusive manner. In 2003, Joung proposed a especially designed group quorum system for group mutual exclusion named the surficial system. Given the total number of manager processes in the system $n$ and the number of groups sought $m$, the degree of the surficial system is $k = \sqrt{\frac{2n}{m(m-1)}}$, which means that up to $k$ processes can be in their critical section simultaneously. In this paper, we propose a new group quorum system for group mutual exclusion of degree $k' = 1 + \sqrt{1 + \frac{n}{m}}$, which is much higher than $k$. Also, when $k = k'$, our system produces quorums of smaller size. This makes our system far more efficient and practical.

1 Introduction

Mutual exclusion is a fundamental problem in distributed systems. In this problem, access to a shared resource by concurrent processes must be synchronized so that only one process can use the resource at a time. The group mutual exclusion problem is a generalization of the ordinary mutual exclusion problem where each application process can belong to different groups and members of the same group are allowed simultaneous access to their critical sections. Members of different groups must access their critical sections in a mutually exclusive manner.

Solutions for group mutual exclusion in shared memory models have been proposed in [2], [6], [10], [11]. In this paper, we consider processes that communicate via message passing. Solutions for group mutual exclusion in such networks have been proposed in [4], [15], [16].

Quorum-based solutions have been proposed for solving both ordinary and group mutual exclusion problems [1], [3], [5], [12], [13], [14]. The basic idea of this type of algorithms is to rely on a set $M$ of manager processes to control access to the critical section. An application process that wishes to enter the critical section has to collect enough votes (permissions) from manager processes to form a quorum $Q \subseteq M$. Under the assumption that each manager process gives out its permission to at most one process at a time\(^1\) (as in Maekawa’s

\(^1\) This assumption is made throughout the paper.
algorithm \cite{12}, if quorums are made such that \( \forall Q_i, Q_j \subseteq M, Q_i \cap Q_j \neq \emptyset \), then mutual exclusion is automatically guaranteed. It is known that quorum-based algorithms are resilient to node failures and network partitioning.

In 2003, Joung proposed a especially designed group quorum system for group mutual exclusion named the surficial system \cite{8}. The main advantage of such approach is that it provides a way from which truly distributed quorum-based solutions for group mutual exclusion can be easily constructed \cite{12}. Moreover, taking a purely mathematical perspective, we think that group quorum systems are an elegant object which study is well justified.

In an \( m \)-group quorum system over a set of manager processes \( P \), there will be \( m \) groups (sets), each of which is a set of quorums of \( P \) such that every two quorums of different groups intersect. Intuitively, an \( m \)-group quorum system can be used to solve group mutual exclusion as follows: Each process \( i \) of group \( j \), when attempting to enter its critical section, must acquire a quorum \( Q \) it has chosen from that group by obtaining permission from every member of \( Q \). Upon exiting the critical section, process \( i \) returns the permission to the members of \( Q \). By the intersection property, no two processes of different groups can enter their critical sections simultaneously.

Under the assumption that each manager process gives out its permission to at most one process at a time, a group quorum system has a main disadvantage: it limits the level of concurrency allowed by the system. This is true because the number of processes in a group that can enter their critical sections simultaneously is bounded above by the maximum number of pairwise disjoint quora in that group (also called the degree of the group). The theoretical upper bound on that degree, provided \( m > 1 \), is \( OPT = \sqrt{n} \), where \( n \) is the number of manager processes in the system \cite{8}. So, under the condition that no more than \( \sqrt{n} \) processes of any group are to enter their critical sections simultaneously, group quorum systems are applicable. Now, if this is unacceptable for some applications, then by relaxing the above assumption, it is not difficult to equip a group quorum system with additional algorithms so that there is no limit on how many processes of the same group can be in their critical sections simultaneously. Such algorithms are described in \cite{8}.

Group quorum systems of optimal degree \((= \sqrt{n})\) are described in \cite{9}. Unfortunately, the work in \cite{9} requires \( n \) to be of the form \( x^2 \), where \( x \) is a power of a prime. To the author’s knowledge, the only known group quorum system that works for any \( n \) and \( m > 1 \), named the surficial system, is described in \cite{8}.

The surficial system has degree \( k = \sqrt{\frac{2n}{m(m-1)}} \), for some integer \( k \), and produces quorums of size \( s = (m-1)k \). In this paper, we describe a new group quorum system based on the notion of Cohorts, a concept introduced by Huang et al. in 1993 \cite{7}. Our system has degree \( k' = 1 + \sqrt{1 + \frac{\sqrt{m}}{m}} \), for some integer \( k' > 2 \), which is much higher than \( k \) for \( m > 3 \). Moreover, the quorum size in our system is \( s' = m(k' - 2) \), which is smaller than \( s \) for the same degree \( k' = k \) and for \( m > k/2 \). These improvements are significant because the degree of a system is related to its fault-tolerance and the level of concurrency it allows. The quorum size is related to the lower bound on the cost of communication imposed on