1. INTRODUCTION

A relevant feature of land use planning is the design of transportation networks.

The problem which public administrators are often confronted with is that of apportioning a limited budget to the various branches of a network so that to achieve the goal put forward by the administration. Namely, given the demand for transportation among the centers connected by the network, one may wish to find the optimal investment policy, in order to minimize the total transportation time.

The main difficulty of this kind of problems is that the objective function turns out to be neither convex nor separable whereas the constraints turn out to be non linear. Some authors have tackled the problem using, in our opinion, too many simplifying assumptions. For example, the transit time on each branch of the network is assumed to be independent of the flow (1) (2) (3). Moreover, in case of several origins and destinations only zero-one investments in each branch are considered (1) (2), whereas discrete investments are analyzed for one origin only (3).

Although the hypothesis that the transit time is independent of the flow seems to be quite coarse, we keep it in this paper, where, as our first contribution to the problem, we propose to overcome all other restrictions. Namely, we consider the case of several origins and destinations with the investments in each branch taken as continuous variables. We also suggest a heuristic computational procedure which appears to be more efficient compared to those so far appeared in the literature.

2. DEFINITION OF THE PROBLEM

The basic hypotheses of this work can be summarized as follows:

**Hypothesis 1**: (transit time hypothesis): the transit time $t_z$ on each arc $z$ is independent of the flow.

**Hypothesis 2** (behavioural hypothesis): the traffic flow distribution is such that the overall transportation time is minimized.

**Hypothesis 3**: the dependence of the transit time $t_k$ associated with arc $k$ on the apportionment $c_k$ is of the following linear type with saturation

$$t_k = a_k - \beta_k c_k \quad 0 \leq c_k \leq c_k^*$$
$$t_k = a_k - \beta_k c_k \quad c_k \geq c_k^*$$

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where
\[ \alpha_k, \beta_k, \gamma_k \] are non-negative constants.

Then the problem can be stated as follows:

**Given:**

1. A transportation network \( G(N,A) \), where \( N \) is the set of nodes and \( A \) the set of arcs.
2. The transportation demand matrix \( R = \{r_{ij}\} \)
3. A budget \( B \)
4. An objective function \( T \)

\[ T = \sum_{i \in N} \sum_{j \in N} t^{T}(c)A_{ij}x_{ij}(c) \]  

(i.e. the overall transit time),

where

\[ x_{ij} : \text{column vector, whose components } x_{ij}^k \text{ represent the flow on the } k\text{-th path } P_{ij} \]

between \( i \) and \( j \), \( i,j \in N \) due to the demand \( r_{ij} \), \( k=1,2,\ldots,q_{ij} \)

\[ q_{ij} : \text{number of different paths between the nodes } i \text{ and } j, \ i,j \in N \]

\[ p_{ij}^k : \text{set of branches in the path } P_{ij}^k \]

\[ A_{ij} : \text{incidence matrix arcs-paths for the origin-destination pair } i-j, \ i,j \in N, \]

whose entries \( a_{m,k}, \ m \in A, \ k=1,2,\ldots,q_{ij} \), are defined by

\[ a_{m,k} = \begin{cases} 1 & \text{for } m \in p_{ij}^k \\ 0 & \text{for } m \notin p_{ij}^k \end{cases} \]  

\( t(c) : \text{column vector with components } t_m(c_m), \ m \in A \)

\( c : \text{column vector with components } c_m, \ m \in A \)

**Minimize** \( T \), subject to the following constraints

\[ \sum_{k=1}^{q_{ij}} x_{ij}^k(c) = r_{ij} \quad \forall i \in N, \forall j \in N \tag{3} \]

\[ \sum_{m \in A} c_m \leq B \tag{4} \]

\[ c \geq 0 \tag{5} \]

\[ c \leq \bar{c} \tag{6} \]