1. Introduction.

The purpose of this paper is to discuss certain new aspects in the theory of growth functions. As regards general introduction to Lindenmayer systems (or, shortly, L systems), we refer the reader to [4], [7] or [8]; or to [10] for the very basic notions only. The basic facts concerning growth functions can be found in [6]. We repeat here only some of the most fundamental notions.

By definition, a DOL system is a triple $G = (V, w, h)$, where $V$ is an alphabet, $h$ is a homomorphism on $V^*$, and $w$ is a word over $V$. The fact that $h(a) = x$, for $a \in V$, is often denoted by $a \rightarrow x$ and referred to as a production. The growth function associated to $G$ is defined by

$$f(n) = \lg(h^n(w)), \ n \geq 0.$$ 

Thus the systems

$$\left(\{a, b\}, a, \{a \rightarrow ab, b \rightarrow b\}\right), \left(\{a\}, a, \{a \rightarrow a^2\}\right), \left(\{a, b, c\}, a, \{a \rightarrow abc^2, b \rightarrow bc^2, c \rightarrow c\}\right), \left(\{a, b, c, d\}, a, \{a \rightarrow abd^6, b \rightarrow bcd^{11}, c \rightarrow cd^6, d \rightarrow d\}\right)$$

have the associated growth functions $n+1$, $2^n$, $(n+1)^2$ and $(n+1)^3$, respectively. A DOL system is propagating (or a P\(\text{DOL}\) system) if $h$ is nonerasing. All our examples above are P\(\text{DOL}\) systems.

A typical production in a DOL system is $b^a \rightarrow x$, meaning that $a$ is rewritten as
x, provided the left neighbour of a is b. Similarly, a typical production in a D2L system is \( b, a, c \rightarrow x \). D1L and D2L systems are special cases of L systems with cell interactions.

The growth function of a D0L system can be given the following matrix representation:

\[
f(n) = \pi M^n \eta,
\]

where \( \pi \), \( M \), and \( \eta \) are a row vector, square matrix and column vector with dimension equal to the cardinality of the alphabet. The entries of \( \pi \) indicate the number of occurrences of each letter in the axiom \( w \). The \((i,j)\)th entry in \( M \) equals the number of occurrences of the \( j \)th letter \( a_j \) in \( h(a_i) \). All entries of \( \eta \) equal 1. Thus, entries of \( \pi \) and \( M \) are arbitrary nonnegative integers. If the same is extended to concern entries of \( \eta \), we get the growth functions associated with the so-called HD0L systems, i.e., systems where another homomorphism \( h_1 \) is applied to each word generated by a D0L system. Obviously, these functions coincide with the \( N \)-rational functions.

The matrix representation implies several useful mathematical properties by which one can get solutions or partial solutions to the problems of analysis (given a system, one has to determine its growth function), synthesis (given a function, one has to realize it, if possible, as a growth function), and equivalence (given two systems, one has to determine whether their growth functions are the same). In particular, D0L (and HD0L) growth functions are always exponential, polynomial or a combination of the two. The following result, [6], is often applied to show that a given function \( f(n) \) is not an HD0L growth function.

**Lemma of long constant intervals.** No function \( f(n) \) having the property that for every integer \( n \) there are integers \( m \) and \( i > n \) such that

\[
f(m+i) \neq f(m+n) = f(m+n-1) = \ldots = f(m)
\]

is an HD0L growth function.

No mathematical characterization corresponding to the matrix representation is known for growth functions of systems with interactions. Accordingly, the results in this area are mainly negative (undecidability) or constructions of some examples. Our paper does not discuss this area at all, and the reader is referred to