A partially ordered set \( T = (T, <) \) is an \( \omega \) - tree, if for each \( x \in T \), the set \( \hat{x} = \{ y : y \in T \land y < x \} \) is linearly ordered by \( < \) and finite. A subset \( X \) of \( T \) is said to be bounded if there exists a natural number \( n \) such that for each \( x \in X \), the set \( \hat{x} \) has at most \( n \) elements. Let \( P_{b1}(T) \) be the set \( \{ X : \text{card}(X) \leq n \land X \text{ is a bounded subset of } T \} \). We take a monadic second-order language \( L \) with one binary relational symbol \( < \) and define the monadic second-order theory of \( T \), denoted by \( \text{Th}_2(T) \), by:

\[
\text{Th}_2(T) = \{ \phi : \phi \text{ is a sentence of } L \land T \vDash \phi \}
\]

We get the weak monadic second-order theory of \( T \), denoted by \( \text{Th}_2^f(T) \), if we restrict the interpretation of the set variables to finite sets only. Similar we define \( \text{Th}_2^b(T) \) (\( \text{Th}_2^{b1}(T) \) respectively) restricting the interpretation of the set variables to bounded sets (elements of \( P_{b1}(T) \) respectively) only. The elementary theory of \( T \) we shall denote by \( \text{Th}(T) \). Let \( K \) ba the class of all \( \omega \) - trees. \( \text{Th}_2(K) \) is defined to be

\[
\bigcap_{T \in K} \text{Th}_2(T).
\]
In a similar way we define $\text{Th}_{2f}(K)$, $\text{Th}_{2b}(K)$, $\text{Th}_{2b1}(K)$ and $\text{Th}(K)$.

Rabin [6] proved that the monadic second-order theory of two successor functions is decidable. From this result he got the decidability of $\text{Th}_2(K')$ and the decidability of $\text{Th}_{2f}(K)$ by a simple interpretability argument (see [6,11]). Here $K'$ is the class of all countable $\omega$-trees.

The following definition we take from [10].

For a model $M$ with relations only, let $M^\#$ be the following model:

(i) its universe is the set of finite sequences of elements of $M$;
(ii) its relations are

(a) $\prec$, where $\bar{a} \prec \bar{b}$ means $\bar{a}$ is an initial segment of $\bar{b}$,
(b) for each $n$-place predicate $R$ from the language of $M$,

$$R_{M^\#} = \{(a_1, \ldots , a_{m-1}, b^1), \ldots , (a_1, \ldots , a_{m-1}, b^n) : a_i, b^i \text{ are elements of } M, M \models R(b^1, \ldots , b^n)\}.$$  

In [10] is given the following result.

**Theorem 1 (Shelah, Stup).** $\text{Th}_2(M^\#)$ is recursive in $\text{Th}_2(M)$.

This result is proved by Shelah and Stup (see [10]) using a generalization of Rabin's automaton from [6]. It implies immediately the decidability of $\text{Th}_2(K)$ (see [10,11]).

The problem of the decidability of $\text{Th}_{2b}(K)$ is raised by Ziegler. He asks whether the theory of uniform Hausdorff-spaces in the language $L_u$ is decidable (see also [4]) and proves the following result, which connects both problems.