OFF-SHELL EFFECTS IN THE NUCLEON-NUCLEON SYSTEM

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OFF-SHELL INFORMATION: Qualitative

When studying the bremsstrahlung reaction

$$A + B + C + D + \gamma$$

we shall suppose that the scattering amplitude for the corresponding nonradiative process

$$A + B + C + D$$

is already known. The latter will sometimes be referred to as "elastic" scattering, but it is not necessary that the final particles (C and D) be the same as the initial ones (A and B). For particles with spin, this requires a knowledge of each of the independent spin amplitudes, although for some purposes, lesser information - such as the unpolarized cross section - may suffice. It will also be assumed that the static electromagnetic moments of the particles A, B, C, and D are known. Together with nonradiative amplitude(s), this will be referred to as "on-shell information". The question one would like to be able to answer about the bremsstrahlung reaction, is this: what new information does it provide? In terms of the matrix element for process (1),

$$\langle \Psi_f^{(-)} | H_{em} | \Psi_i^{(+) \rangle}$$

this information can be of two quite different types. First, is the behaviour of the wave functions in the non-asymptotic region, or equivalently, the off-shell (strong interaction) amplitude. Second, is the departure of $H_{em}$ from the electromagnetic interaction of the free particles; this is also called "exchange currents".

The off-shell amplitude appears most simply in the external emission diagrams, shown on the lower portion of Fig.1. In diagram A, for example, particle 1 emits the photon before any of the strong interaction takes place. The circle on the figure is called the off-shell amplitude. When dealing with Feynman diagrams, it is an off-mass-shell amplitude; with Lippmann-Schwinger diagrams it is an off-energy-shell amplitude. It is important to note that the latter is not a covariant object, even though it can be an ingredient of a theory which yields Lorentz invariant answers for observable quantities, such as the bremsstrahlung cross section. All other contributions to the bremsstrahlung amplitude are called internal emission.

It is intuitively clear that if the photon energy is too small, there cannot be any significant off-shell information to be obtained. But how small is too small? One of the relevant parameters is the product of the frequency of the radiation $\omega$ with the duration of the collision $\tau$. The argument from classical
Fig. 1. The four off-energy-shell T matrix elements associated with a single bremsstrahlung event (momentum transfer is not shown). The insert shows the kinematics of this UCLA coplanar ppy event. For diagram A, for example, $q_{on}$ is the magnitude of the (relativistic) relative momentum of the protons with momenta $p'_1$ and $p'_2$; and $q_{off}$ is the same quantity for the protons with momenta $p_1 - \vec{E}$ and $p_2$. $q_{off}$ is calculated in the laboratory frame. The point labelled L is the common one about which Low expanded all the T-matrices. Feshbach and Yennie expanded the T-matrices of diagrams A and C about the point $F-Y_f$, and those for diagrams B and D about $F-Y_i$.

Radiation theory, which was presented in 1937 by Nordsieck, goes as follows. The amplitude of the radiation field with frequency $\omega$ is proportional to

$$e \int_{-\infty}^{\infty} dt e^{-i\omega t} \vec{r}_\perp(t)$$

where $\vec{r}_\perp(r)$ is the component of the particle's acceleration perpendicular to the direction of propagation of the radiation. If $\omega t << 1$, the exponential can be replaced by unity, and the answer is just $(\vec{v}_f - \vec{v}_i)_\perp$. [The same reasoning can be applied relativistically, and results in each $\vec{v}$ being replaced by $\vec{v}/(1 - \vec{v} \cdot \hat{k})$.] In other words, if $\omega t << 1$ the radiation is completely determined by the elastic