Geometrically Formulated Gauge Dynamics for Extended Hadrons

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Abstract:
A de Sitter gauge invariant set of field equations is investigated as a possible basis for a gauge description of extended hadrons. The formalism uses an underlying geometric structure given by a fiber bundle over space-time with Cartan connection possessing as fiber a 4-dimensional space of constant curvature characterized by a curvature radius $R$ chosen to be of the order of a Fermi. The constant $R$ represents an elementary length parameter of geometric origin associated with strong interaction physics. A curvature is induced on the bundle space through a hadronic matter distribution described by a generalized bilocal wave field $\psi(x,\xi)$ where $x$ denotes a point in the base space (space-time) and $\xi$ varies in the local fiber. An expansion of the internal motion associated with the variable $\xi$ is given in terms of "de Sitter plane waves", i.e. the so-called horospherical waves, which are the analogue of the usual plane waves in flat Minkowski space-time. In this context the harmonic analysis of scalar and spinor fields in $(4,1)$ de Sitter space is discussed and its relevance to the $SO(4,1)$ gauge theory is pointed out.

I. Introduction
To gain an understanding of the intriguing problem of hadron structure and the hadronic mass spectrum it seems essential to incorporate into the theoretical framework the fact that the hadrons observed in nature are extended objects. We want to take this property from the very beginning into consideration by introducing at a fundamental level of the description an elementary length parameter $R$ of the order of one Fermi. To define a geometric stratum on which extended hadrons could manifest themselves as extended entities we shall assume that in the small the space to be used in the formulation of a hadron dynamics possesses a richer structure than the one given by a four dimensional flat space-time manifold. In fact, we shall assume that the basic geometry to be used in representing hadronic phenomena is provided by a fiber bundle with Cartan connection raised over space-time and possessing the $SO(4,1)$ de Sitter group as structural group. Without repeating the motivation for this choice in detail we mention four relevant points*):

i) An elementary length is built into the underlying geometric structure in characterizing the fiber of the bundle by a length parameter $R$. As fiber we take a 4-dimensional space-time of constant curvature, $\mathbb{V}_4$, with curvature radius $R$ (being, as mentioned, of the order of a Fermi) on which the $SO(4,1)$ de Sitter

*) Compare in this context the arguments presented in refs. 1-3.

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group*) acts as a group of motion. \( V'_4 \) is isomorphic to the noncompact coset space \( SO(4,1)/SO(3,1) \).

ii) The Cartan nature of the bundle implies that the fiber over each space-time point \( x \), i.e. the local de Sitter space \( V'_4(x) \), is tangent to space-time at \( x^{**} \).

iii) The suggested geometric framework allows a hadronic matter distribution, represented by a bilocal wave field \( \psi(x,\xi) \) with \( x \in V'_4 \) and \( \xi \in V'_4(x) \), to react back on the geometric stratum by inducing a curvature on the bundle space which is determined by a hadronic current associated with the field \( \psi(x,\xi) \).

iv) The Casimir operators of the de Sitter group correlate mass and spin leading to a rotator spectrum of states similar to a Regge spectrum with the higher mass states possessing larger spins**(4)****).

The geometrically motivated formalism leads to a gauge description for extended yet elementary hadrons**(1-3)**(7). There are no constituents in the true sense of the word introduced in this description. Hadronic matter is represented, as already mentioned, by a generalized bilocal wave field \( \psi(x,\xi) \) with \( x \) labelling a space-time point and the internal coordinates \( \xi \) varying in the local fiber, \( V'_4(x) \), the latter being isomorphic to a copy of an abstract \((4,1)\) de Sitter space sitting over the point \( x \). Technically speaking \( \psi(x,\xi) \) is a cross section on a vector bundle associated with the de Sitter frame bundle, the latter being a principal fiber bundle over space-time with the fiber and structural group being identical to the \( SO(4,1) \) de Sitter group. The section \( \psi(x,\xi) \) will carry representation character with respect to the local Lorentz group operating in \( T_x \), the tangent space to \( V_4 \) at \( x \), and with respect to the internal or gauge group \( SO(4,1) \). We shall factor \( \psi(x,\xi) \) into \( \psi(x,\xi) = \varphi(x) \cdot \psi_x(\xi) \) where our present knowledge requires – as we shall briefly indicate below – to regard the spacetime part \( \varphi(x) \) as a local \( q \)-number field of conventional quantum field theory, whereas the de Sitter part \( \psi_x(\xi) \) is a \( c \)-number field describing the motion in the fiber over \( x \). \( \psi_x(\xi) \) is a quantity varying smoothly in \( \xi \) and \( x \) on the bundle space.

The use of a generalized wave operator \( \psi(x,\xi) \) with specific \( q \)-number and \( c \)-number content amounts to the description of hadronic phenomena in terms of a bilocal field (or generalized wave function in a one-particle theory) being observable only modulo the action of a gauge group \( G \) which for physical and geometrical reasons is here taken to be the noncompact ten parameter group \( SO(4,1) \). The freedom of changing the local

*) We take \( SO(4,1) \) in favour of \( SO(3,2) \) since the associated \((4,1)\) de Sitter space is compact in its spacial extensions and noncompact in time, while the \((3,2)\) de Sitter space has closed timelike geodesics.

**) The tangent space \( T_x \) to space-time at \( x \) and the tangent space \( T_\xi \) to \( V'_4(x) \) at \( \xi \) are identified by an isomorphism (soldering condition). \( \xi \) is the point left invariant by the Lorentz subgroup \( SO(3,1) \) of \( SO(4,1) \).

***) \( V_4 \) denotes a generally curved Riemannian space-time. For most of the subsequent discussion we shall, however, disregard long range gravitational fields and assume the base space to be flat Minkowski space-time \( M_4 \).

****) Compare also ref. 5 and 6 in this context.