AN ARBITRARY PRECISION
REAL ARITHMETIC PACKAGE IN REDUCE*)

Tateaki SASAKI

Department of Computer Science
The University of Utah
Salt Lake City, Utah 84112, USA

ABSTRACT

A REDUCE arbitrary precision real arithmetic package is described which will become a part of the kernel of an algebraic-numeric system being developed for REDUCE. The basic design principles of this package are first, it is as efficient as possible in both calculation speed and memory usage, second, even a casual user can use it, and third, it is highly portable and extensible. Our idea to attain the first property is to represent the arbitrary precision real number in as short a form as possible and to handle the precision in a much more flexible manner than any other similar system. A comparison is made of our scheme with a conventional one which uses a global precision, verifying the efficiency of our scheme. Our package contains two sets of routines for elementary arithmetic operations such as addition or multiplication. An expert user can write efficient programs using the first set of routines, while a casual user may use the second set of routines with less programming effort. Our package will become faster by rewriting only four basic and simple routines machine-dependently.

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1. Introduction

The present algebraic and symbolic systems are basically designed for algebraic and symbolic calculations and are not well suited for numeric calculations. However, there is a clear need for systems which enable the user to use algebraic and numeric methods concurrently. For many problems, numeric and algebraic methods provide us different approaches which often supplement each other. Furthermore, interactive usage of algebraic and numeric methods is often very effective for many complicated problems. This point has been stressed recently in a survey paper by Brown and Hearn [1]. With this in mind, a complete numeric support package is being developed by a collaborative effort between the University of Utah and the Institute of Physical and Chemical Research, Japan. The arbitrary precision real arithmetic package (to be called the big-float arithmetic package hereafter) to be described here is a part of this system, although the package is complete by itself.

An algebraic-numeric system must be equipped with a big-float arithmetic module. One reason for this is that fixed precision real arithmetic, as in FORTRAN, is not sufficient to evaluate long or non-regular expressions for which algebraic systems are best suited to manipulate. Second, many of the algebraic system-oriented numerical algorithms require big-float arithmetic. Some typical examples are algorithms given by Richman [2] and Pinkert [3].

Because of its importance, big-float arithmetic was implemented in several symbolic systems, in SAC-I by Pinkert [4], in MACSYMA by Fateman [5], and in HLISP by Kanada [6]. Furthermore, there are a number of FORTRAN-based systems for big-float arithmetic [7,8]. However, our package is somewhat distinct from these systems.

The greatest difference is in the handling of the numerical precision. Since calculations of big-float numbers in symbolic systems are expensive in both time and memory, we intend to make our package as efficient as possible in both speed and memory usage. Our idea to attain efficiency is to represent the big-float number in as short a form as possible and to treat the precision in a much more flexible manner than any other system constructed so far. This sophistication of the package increases the programming task of the user because he must control the precision for himself. We therefore provide another set of arithmetic operation routines in which the precision is controlled automatically. In addition to these features, much emphasis is placed on high portability and extensibility of the package. This is because that we intend to make the package not only a part of the kernel of our algebraic-numeric system but also a big-float arithmetic module for many LISP systems.