HYPERPARAMODULATION: A REFINEMENT OF PARAMODULATION

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Abstract

A refinement of paramodulation, called hyperparamodulation, is the focus of attention in this paper. Clauses obtained by the use of this inference rule are, in effect, the result of a sequence of paramodulations into one common nucleus. Among the interesting properties of hyperparamodulation are: first, clauses are chosen from among the input and designated as nuclei or "into" clauses for paramodulation; second, terms in the nucleus are starred to restrict the domain of generalized equality substitution; third, total control is thus iteratively established over all possible targets for paramodulation during the entire run of the theorem-proving program; and fourth, application of demodulation is suspended until the hyperparamodulant is completed. In contrast to these four properties which are reminiscent of the spirit of hyper-resolution, the following differences exist: first, the nucleus and the starred terms therein, which are analogous to negative literals, are determined by the user rather than by syntax; second, nuclei are not restricted to being mixed clauses; and third, while hyper-resolution requires inferred clauses to be positive, no corresponding requirement exists for clauses inferred by hyperparamodulation.

To illustrate the value of this refinement of paramodulation, we have chosen certain conjectures which arose during the study of Robbins algebra. A Robbins algebra is a set on which the functions $o$ and $n$ are defined such that $o$ is both commutative and associative and such that for all $x$ and $y$ the following identity
$$n(o(n(o(x,y)),n(o(x,n(y)))))) = x$$
holds. One may think of $o$ as union and $n$ as complement. The main interest in such algebras arises from the following open question: If $S$ is a Robbins algebra, is $S$ necessarily a Boolean algebra? The study of this open question entailed heavy use of an automated theorem-proving program to examine various conjectures. Certain computer proofs therein were obtained only after recourse to hyperparamodulation. (These lemmas were actually obtained prior to the work reported on here by Winker and Wos using a non-standard theorem-proving approach developed by Winker.)

Section 1. Introduction

In this paper we discuss the inference rule, hyperparamodulation, which is a refinement of paramodulation. We give some detail concerning its implementation, the motivation for its formulation, and the way it is used. We make comparisons of it to both paramodulation and hyper-resolution in terms of its properties and effectiveness. Since hyperparamodulation requires the user to select from among the input clauses those into which so-called equality

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substitution is permitted, a heuristic is provided for making that choice. Similarly, since
the user also designates within the chosen input clauses which terms are admissible for
replacement, we suggest a heuristic for that purpose also.

We then turn in Section 4 to an example from algebra, specifically from Robbins algebra,
for a detailed examination of the kind of proof most suited to the use of hyperparamodula-
tion. Since this proof itself, the study from which it is taken, and the set of experiments
underlying this paper all implicitly reflect certain views of the authors, we make those
views explicit in Section 2. We also discuss there certain additional procedures and their
effect on program and user complexity.

Although this study is a very preliminary study, the evidence appears to be of some
significance. We believe that the appropriate purpose of workshops is to encourage the
dissemination of such studies and experience. We welcome any evidence even remotely germane
to any position taken herein.

Section 2. Tacit Assumptions

2.1. The Program as a Research Tool

One use of the automated theorem-proving program implemented at both Argonne National
Laboratory and Northern Illinois University is that of attacking open questions in mathem-
atics. An example of this is provided by our study of Robbins algebra. In attempting to
answer the open question given in Section 4, a number of conjectures were made about possible
lemmas. Each was submitted to the theorem-proving program with the proviso that no special
programming features be added or employed, but also with the proviso that the user could
adjust any or all of various input parameters to reflect his preference. We note first that,
in most cases, the program did generate a proof using hyperparamodulation. Second, it was
not necessary to change parameter settings from the defaults for any of the lemmas tried.
Most significantly, the program was not able to get proofs using paramodulation alone. A
number of these conjectures are still open, and in particular, the main question is still
unanswered. No attempt was made to have the program produce a counter-example to any
conjecture, although this capability is available in part.

At this point, it is natural for three questions to be asked. Was all of the research
done with computer runs? If a proof was not found, wasn't one then in a quandary? If one
can set various parameters at will, doesn't that give the set of experiments an ad hoc
flavor?

Aside from the user making the conjectures about lemmas, the answer to the first question
is, essentially, yes. If a proof was obtained from the first attempt, it was catalogued, and
the next conjecture was tackled. If no proof was found, various parameters were changed to
permit a wider and somewhat different search. In addition, sometimes lemmas the user judged
worthwhile and which were found in previous runs were added. In most cases, but a few
failures were sufficient to table the conjecture. Occasionally, some results obtained by
hand were adjoined, somewhat in the spirit of using suggestions made by some other
mathematician.

As for the second question, tabling of a conjecture leaves one no worse off than when the
conjecture was made. Put another way, if a colleague fails to make any helpful suggestion,
one still continues to seek his assistance with new conjectures.